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1 **Forecasting the number and distribution of new bidders**
2 **for an upcoming construction auction**

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5 **Abstract**

6 Estimating the number of new bidders in construction auctions is relevant for both
7 private companies and contracting authorities. For private companies, it allows the total
8 number of competing bidders to be estimated which may lead to better adjustments of future
9 bids. For contracting authorities, it allows the population size of all potential bidders' to be
10 estimated and thus to implement better awarding criteria. Mathematical models for forecasting
11 the number of new bidders and the population size of all potential bidders are, however, very
12 scarce in the construction management literature.

13 In this paper, we propose an Exponential model for predicting the average number of
14 new bidders based on an urn analogy. The model allows the number of new bidders to be
15 estimated as a function of new versus total participating bidders observed in previous auctions.
16 The parameter estimates obtained from the model also allow the statistical distribution of the
17 number of potential new bidders to be modelled using a sum of Binomial distributions. We
18 validate the Exponential model on three published construction auction datasets, showing that
19 the proposed model significantly outperforms the most advanced model for performing similar
20 tasks – the Multinomial model proposed by Ballesteros-Pérez & Skitmore (2016).

21 **Keywords:** bidders, auction, forecasting, exponential model, binomial, competitiveness.

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22 **Introduction**

23 A ubiquitous feature of the construction industry worldwide is the use of tendering for
24 the simultaneous selection of contractors and soliciting a competitive price. This generally
25 involves a sealed bid auction, in which the number of bidders is taken to be a proxy of the level
26 of competition in the market. It is not surprising therefore that both the number and the identity
27 of potential competitors are among the most relevant factors when a bidder is deciding whether
28 to submit a bid or not (Ahmad and Minkarah 1988; Shash 1993).

29 The number of bidders and their dominant competitive profiles greatly condition
30 several auction outcomes (e.g. Dyer, Kagel, & Levin 1989; Hu, 2011; Levin & Ozdenoren
31 2004; Takano, Ishii, & Muraki 2014). The winner's curse, in which an unaware bidder submits
32 an abnormally low bid that is eventually awarded, is one of the most celebrated (Capen et al.
33 1971). Indeed, research confirms that while experienced bidders are more successful on
34 average (Fu, Drew, & Lo 2002, 2003), inexperienced bidders are more prone to submit
35 abnormally high or low bids (Ballesteros-Pérez et al. 2015b).

36 All these facts are clearly relevant to competing bidders in assessing a more realistic
37 level of competition and their competitors' pricing strategies. But they are also relevant to
38 owners in designing their awarding criteria (Liu et al. 2015), as those that restrict market entry
39 make markets less efficient (the opportunity cost is greater, with prices higher than in perfect
40 competition). On the other hand, awarding contracts to bidders who cannot cover their costs
41 may create serious problems down the production line, and may even lead to project failure.

42 Bidding models have been developed to aid decision making by both owners and
43 bidders. They are usually statistical in nature, making full use of random variables as a
44 simplification mechanism by means of agglomerating the considerable uncertainties involved
45 in construction tendering. A fundamental simplification applied to many theoretical treatments

46 is to avoid the complexities involved in having to deal with different bidders' profiles by
47 assuming that bidders' bids are independently and identically distributed (*iid*) (see Klemperer
48 (2004) for a review of the main contributions). However, empirical studies of construction bids
49 by Oo, Drew, & Lo (2010) and Skitmore (1991) have demonstrated the untenability of *iid* in
50 real construction auctions.

51 Other, more serious, Bid Tender Forecasting Models (BTFMs) analyze bidders'
52 competitive profiles separately (e.g. Pablo Ballesteros-Pérez, González-Cruz, & Cañavate-
53 Grimal 2013; Carr 1982; Friedman 1956; Gates 1967; Skitmore & Pemberton 1994). This is
54 done from historical auction data, while bidders with no past record are modeled as "average"
55 bidders. However, quantifying the *number* of these new bidders *in advance* of the auction has
56 other problems, such as inferring their potential participation when there is no previous
57 available information about them (Runeson and Skitmore 1999).

58 The result is that BTFMs normally work well in auction datasets with a high proportion
59 of regular bidders. Unfortunately, this is again not the case in the construction industry, where
60 there is usually a large population of bidders with varied and partially overlapping areas of
61 expertise, and where an irregular few bid or are selected to bid (Skitmore 2013b). BTFMs that
62 take into consideration different bidding profiles, despite being more information-demanding,
63 are generally the most accurate (Ballesteros-Pérez et al. 2016a). These BTFMs can vary in
64 complexity, from considering just two competitive bidding profiles (e.g. regular vs new
65 bidders) to treating every single bidder's identity separately (Ballesteros-Pérez et al. 2016c).
66 Irrespective of how many and which bidding profiles are considered, however, they all need to
67 quantify how many bidders of each bidding category (profile) will submit a bid. Without this
68 information and considering the number of new bidders is a significant proportion of the total
69 number of bidders as discussed earlier, these BTFMs cannot produce reliable estimates.

70 However, apart from Mercer & Russell's (1969) (unsuccessful) attempt to infer the
71 appearance of new bidders from the periodicity of bid submissions of frequent bidders, and
72 Ballesteros-Pérez & Skitmore's (2016) recent Multinomial model, no other methods have been
73 published for predicting the number of new bidders.

74 In this paper, we propose an Exponential model for anticipating the number of new
75 participating bidders in an upcoming auction based around the Binomial distribution. We
76 provide not only the statistical distribution of the number of new bidders but also an improved
77 estimate of the size of the population of potential bidders. We estimate the model on three sets
78 of construction bidding data and find the proposed model to be twice as accurate as the
79 Multinomial model, the only previous known model available. The approach suggested in this
80 paper is relevant to practice for both open and selective tendering schemes.

81 This paper is structured as follows. In the *literature review* section, we briefly
82 summarize the major works on anticipating the number of bidders and the population size of
83 potential bidders. Due to its relevance, the Multinomial model, the only existing model for
84 anticipating the number of new bidders, is described in detail. In the *materials and methods*
85 section, we describe the proposed Exponential model from an urn analogy, its constituent
86 equations, and how to implement the model for forecasting purposes. In the *model validation*
87 section, we apply the Exponential model to three datasets of construction auctions, and
88 compare the model's performance with that of the Multinomial. A dedicated section on the
89 *distribution of the number of new bidders* follows where we show how the Binomial
90 distribution can be used to estimate the number of new bidders in an upcoming auction. In the
91 *population of bidders* section, we illustrate how the population of bidders grows in the three
92 auction datasets. In the *discussion* section, we consider the practical relevance of the proposed
93 model, as well as how the model may be improved. Finally, the *conclusions* section

94 summarizes the major contributions of the paper, including limitations and avenues for future
95 research.

96

97 **Literature review**

98 The number of *new* bidders participating in an upcoming auction can be inferred from
99 the proportion of past *new* bidders divided by the *total* number of participating bidders, as well
100 as from the population size of potential bidders. However, neither obtaining, nor interpreting
101 these variables turns out to be a trivial task.

102 Friedman (1956) was among the first researchers to suggest that the Poisson
103 distribution may provide a suitable model to estimate the total number of participating
104 bidders. The Poisson distribution depends on a single parameter λ whose best unbiased
105 estimate corresponds to the average number of participating bidders in previous auctions.
106 Much empirical work, however, has found both supportive and unsupportive evidence of
107 such a fit. A further assertion by Friedman is that the Poisson distribution may model the
108 errors (deviations) of the number of bidders instead – both claims being rejected by
109 Skitmore's (1986) empirical analysis of three sets of real UK construction data.

110 Since Friedman's work, a plethora of statistical distributions (e.g. Normal, Log-
111 Normal, Uniform, Weibull, Gamma, and Laplace) have been proposed for modelling the total
112 number of participating bidders (Ballesteros-Pérez, Skitmore, et al. 2015; Engelbrecht-
113 Wiggans, Dougherty, & Lohrenz 1986; Skitmore 2013a; Stark & Rothkopf 1979). In this
114 regard, Ballesteros-Pérez et al. (2015) performed an extensive fit analysis spanning 12 datasets
115 of construction tenders from four continents. They conclude that, on average, the lognormal
116 distribution performs best, closely followed by the Normal, Logistic, and Log-Logistic
117 distributions.

118 Despite the generally poor performance in terms of modelling the number of
119 construction bids, the Poisson model has endured in practice. It is still the most popular model,
120 not just in construction auctions, but also in other settings such as online auctions (Mohlin et
121 al. 2015) and numerical simulations (Takano et al. 2014).

122 Alternative proposals to treat the number of bidders participating in an auction
123 stochastically have been made by Rubey & Milner (1966), who suggested resorting to
124 experience and observation to anticipate the average value of the participants in upcoming
125 auctions. This approach has been refined by many other researchers who confirmed that,
126 indeed, the number of bidders is different depending on other aspects such as project type and
127 size (Azman 2014; Drew and Skitmore 2006), client (Ballesteros-Pérez, González-Cruz,
128 Pastor-Ferrando, & Fernández-Diego 2012), geographical location (Al-Arjani 2002; Benjamin
129 1969) and market conditions (Ngai et al. 2002; Skitmore 1981).

130 However, most of these aspects are quite difficult to standardize (Lan Oo et al. 2007;
131 Oo et al. 2010a; b) and/or are strongly context-specific (their regression relationships remain
132 valid providing the region, client, economic context, and/or the type of projects are similar)
133 (Ballesteros-Pérez et al. 2015a). The only refinement (to having purely random variables) that
134 works in most contexts has been to resort to the contract size (project budget) as a proxy for
135 the total number of participating bidders. In particular, by classifying past auctions into
136 homogeneous categories (same or similar project type, client, and location), this involves
137 exploiting the generally moderate correlation between contract size and the number of bidders
138 to make better predictions of λ (Rickwood 1972; Wade and Harris 1976). However, the
139 proposed model focuses on forecasting the number of new bidders in an upcoming auction, not
140 the total number of participating bidders. The latter has been the subject of other recent analyses
141 by Ballesteros-Pérez et al (2015a). Furthermore, we express the number of new bidders as a

142 proportion of the total participating bidders. This will allow the forecasting of both variables
143 to be treated as separate, independent, problems.

144 Estimating the population size of all potential bidders has proven even more elusive
145 than the number of participating bidders per auction. So far, only Ballesteros-Pérez & Skitmore
146 (2016) have successfully attempted this in an approach proposed in parallel with their
147 Multinomial model. This basically resorts to dividing the total amount of different bidders
148 identified so far (the size of the bidders' identities database) by the proportion of new versus
149 total bidders in the last auction. This estimate provides a reasonably close approximation, but
150 has the disadvantage of suffering high variability. Therefore, a large number of auctions is
151 generally required to obtain accurate estimates.

152 Finally, concerning the number of new bidders, researchers to date have only produced
153 one model: the Multinomial model described in the next subsection. The advanced reader,
154 though, may also think of other alternative routes to derive a relatively good estimate of the
155 number of new bidders in an upcoming auction. For example, subtracting the number of
156 frequent bidders (those who have already been identified in the database) from the population
157 size of all potential bidders, and then estimating how likely it is that a proportion of those will
158 submit a bid again in upcoming auctions. However, this approach has several problems. First,
159 it requires classifying the identities of bidders in relatively homogeneous groups (Ballesteros-
160 Pérez, Skitmore, Pellicer, & Gutiérrez-Bahamondes 2016; Shaffer & Micheau 1971; Wade &
161 Harris 1976). This can be very misleading as bidders may bid for different types of work (multi-
162 market scheme) (Morin and Clough 1969) or stop bidding altogether when all their resources
163 are busy (Lan Oo et al. 2012; Skitmore 1988). As both act in opposite directions, analyzing
164 bidders in homogeneous groups can be very unreliable. Second, it still requires an estimate of
165 the total number of participating bidders in an upcoming auction. So far, there are no reliable
166 models to accomplish this task. There are no other alternatives so far.

167

168 *The multinomial model*

169 Currently, only one model has been proposed to estimate the number of new bidders.
170 This model is an implementation of the multinomial distribution for construction auctions
171 proposed by Ballesteros-Pérez & Skitmore (2016). It contains two variants, a trinomial model
172 for forecasting the total number of different bidders for auction $i+1$ (the current auction is
173 indexed as auction i), and a Binomial model for forecasting the current number of once bidders
174 for auction $i+1$. The difference between the Trinomial and the Binomial models is that when
175 bidders become twice bidders, they are promoted to a different category and are no longer
176 counted the Binomial model, only by the Trinomial model. This is the reason why the
177 Trinomial model will be the one compared later.

178 The Multinomial model works by applying random walks to sets of nonce-, once-,
179 twice-, thrice-bidders, etc. up to the maximum possible number of bids submitted by each
180 bidder up to and including auction i . All subgroups of bidders who have been bidding more
181 frequently in the past are also assumed proportionally more likely to submit another bid in
182 auction $i+1$. Therefore, the Multinomial model basically counts how many once-, twice-,
183 thrice-bidders and so on, have been currently identified in the tender dataset. Next, it estimates
184 how likely it is that each of these bidders will submit another bid, and sums these estimates to
185 provide an overall estimate of the total (probabilistic) number of (new and frequent) bidders.
186 This capability allows the multinomial model to forecast the total number of bidders
187 participating in an upcoming auction and not just the new bidders (although not that
188 accurately). For that purpose, the model resorts to a regression-based estimate of the population
189 size of all potential bidders. This is also possible by the urn model described later. Indeed, we
190 will show that the urn model estimates compare favorably with the Multinomial estimates.

191 The Multinomial model is substantially more mathematically complex than the
192 Exponential model we propose. This is partially unavoidable as the former includes other
193 capabilities that are of no interest when forecasting the number of new bidders. Our analysis
194 here focuses on comparing the Multinomial and Exponential models in terms of predicting the
195 number of new bidders in an upcoming auction as well as in estimating the population size of
196 all potential bidders.

197

198 **Materials and methods**

199 In this section, we present the Exponential model, as well as the representative urn
200 analogy upon which the model is built.

201

202 *Notation*

203 The proposed model makes use of the following terminology, some of which has
204 already been presented:

205 i The i^{th} auction

206 N Total population of all potential bidders.

207 N_i Number of bidders participating in auction i .

208 N_1^{i*} Number of different bidders participating in auctions 1 to i .

209 N_i^* Number of new bidders in auction i (they had not submitted a single bid in auctions 1
210 to $i-1$). It can be calculated as $N_i^* = N_1^{i*} - N_1^{i-1*}$. By definition, $0 \leq N_i^* \leq N_i$.

211

212

213 *An urn analogy*

214 The number and proportion of new bidders found in an auction i (or $i+1$) can be
215 assimilated to an urn containing N different balls (each one representing one bidder). Each
216 auction i is represented by a draw of N_i balls (total number of participating bidders in that
217 auction). After each draw, all balls are returned to the urn and they can be drawn again in future
218 (sampling with replacement after each draw).

219 As successive auctions (draws) take place ($i=1,2,3\dots$), the number of balls that have
220 not been drawn before are quantified as new bidders (N_i^*). If we simulate the proportion of
221 new balls versus the total number of balls drawn, that is N_i^*/N_i , the expected values of these
222 ratios are represented in Figure 1.

223 Figure 1 was generated using 12,000 (Monte Carlo) simulations assuming a population
224 of $N=100$ bidders (balls). Each line represents the successive values of N_i^*/N_i as auctions
225 (trials) progress ($i=1,2,3\dots$) and for different number of bidder (balls drawn) per auction ($N_i=1,$
226 $2, 3, \dots, 100$ bidders).

227 **[Insert Figure 1 here].**

228 Similar results are obtained with alternative population sizes, irrespective of the total
229 number of trials (i) or the size of each draw (N_i). From the graph it appears that the average
230 values of N_i^*/N_i are well represented by an Exponential function. These empirically obtained
231 expressions all have the same generic form if we choose the Euler's number as the logarithmic
232 base:

233
$$\frac{N_i^*}{N_i} = a \cdot e^{b \cdot i} \quad (1)$$

234 where a and b are the two coefficients that define the Exponential regression line.

235 All lines cross the same point ($i=1, N_i^*/N_i=1$), as all bidders (balls) are necessarily new
 236 ($N_i^*=N_i$) in the first auction ($i=1$). By exploiting this boundary condition and taking log values,
 237 one of the regression parameters can be expressed as a function of the other as follows:

$$238 \qquad \qquad \qquad b = -LN a \qquad \qquad \qquad (2)$$

239 where $LN a$ is the natural logarithm of a .

240 Next, by substituting (2) into (1) we obtain:

$$241 \qquad \qquad \qquad \frac{N_i^*}{N_i} = a \cdot e^{-i \cdot LN a} \qquad \qquad \qquad (3)$$

242 Therefore, in terms of bidding, when using expression (3) to forecast the proportion of
 243 new bidders in an upcoming auction, it will be necessary to compute the value of a from
 244 previous auctions. Fortunately, the value of a can be easily obtained from expression (3) taking
 245 log values:

$$246 \qquad \qquad \qquad a = \left(\frac{N_i^*}{N_i} \right)^{\frac{1}{1-i}} \qquad \qquad \qquad (4)$$

247 where N_i^*/N_i is the observed value of new versus total bidders participating in auctions up to i .

248 Inserting expression (4) into (3), the mathematical expression for the proportion of new
 249 bidders in auction $i+1$ is therefore:

$$250 \qquad \qquad \qquad \frac{N_{i+1}^*}{N_{i+1}} = a \cdot e^{(1-i) \cdot LN a} = \left(\frac{N_i^*}{N_i} \right)^{\frac{1}{1-i}} e^{\frac{i+1}{i-1} \cdot LN \left(\frac{N_i^*}{N_i} \right)} \qquad \qquad \qquad (5)$$

251 Analogously, if the total number of participating bidders in auction $i+1$ could be
 252 somehow anticipated, expression (5) can be reorganized to compute, instead of the proportion,
 253 the number of new bidders:

254
$$N_{i+1}^* = N_{i+1} \cdot a \cdot e^{(1-i)LN a} = N_{i+1} \left(\frac{N_i^*}{N_i} \right)^{\frac{1}{1-i}} e^{\frac{i+1}{i-1} LN \left(\frac{N_i^*}{N_i} \right)}$$
 (6)

255 Finally, the previous expressions also allow the population size of potential bidders,
 256 that is N , to be estimated. The exponents of the three regression equations in Figure 1 of 0.01
 257 (for $N_i = 1$), 0.02 (for $N_i = 2$), and 0.03 (for $N_i = 3$) were obtained assuming a bidder population
 258 of 100 ($N=100$), indicating that a , N_i and N are empirically related as follows:

259
$$N = \frac{N_i}{LN a}$$
 (7)

260 Again, this expression is identical for alternative sets of a , N_i and N values in the urn
 261 model. However, despite the fact that expressions (5) to (7) depend on a parameter which is
 262 (theoretically) constant (parameter a can be obtained with expression (4) by means of a single
 263 auction), a more accurate estimate of a can be obtained by analyzing a larger number of
 264 auctions. Similarly, in practice, neither the number of participating bidders (N_i) nor the
 265 population size of all potential bidders (N) are constant for all auctions over long periods of
 266 time. Therefore, it is practical to resort to “average” values of a and N_i to improve the quality
 267 of the estimates of (5) to (7), namely:

268
$$a_{avg} = \frac{1}{i} \sum_i a_i$$
 (8)

269
$$N_{avg} = \frac{1}{i} \sum_i N_i$$
 (9)

270 This leads to the following expressions that can be used for forecasting purposes:

271
$$\frac{N_{i+1}^*}{N_{i+1}} = a_{avg} \cdot e^{(1-i)LN a_{avg}}$$
 (10)

272
$$N_{i+1}^* = N_{avg} a_{avg} \cdot e^{(1-i)LN a_{avg}}$$
 (11)

273
$$N = \frac{N_{avg}}{LN a_{avg}} \quad (12)$$

274 Among other applications described earlier, expression (12) can be used to monitor the
275 time-varying population size of potential bidders (N) as more auctions are completed.

276

277 ***Model validation***

278 We empirically test the proposed Exponential model on three published datasets of
279 construction auctions. The three datasets, containing all economic bids (not used in this study)
280 and the numerical codes representing the bidders' identities, are provided in the *Supplemental*
281 *Online Material* file.

282 Table 1 shows that the major features of the three datasets are vastly different. Dataset
283 1 contains bidder data in the London area over a short period of time. Dataset 2 contains bidder
284 data in a wider area of the north of England for much smaller projects and over a longer time
285 period. Dataset 3 contains bidder data in the highly competitive Hong Kong construction
286 market, where as many as 33 bidders may be bidding for a single contract.

287 **[Insert Table 1 here]**

288 For all tender datasets, we forecast the proportion (when N_{i+1} is assumed known) and
289 the number (when N_{i+1} is assumed unknown) of new bidders in auction $i+1$ using the proposed
290 Exponential model. To make the best forecasts possible for auction $i+1$, all information up to
291 and including auction i is utilized.

292 Table 2 compares the performance of the proposed Exponential model versus the
293 Multinomial model of Ballesteros-Pérez & Skitmore (2016). The detailed auction-by-auction
294 results of the Exponential model can be found in the *Supplemental Online Material* file. For
295 the performance evaluation, the absolute, instead of squared, errors are preferred since absolute

296 errors allow the error magnitude to be expressed in a more meaningful way, i.e. number of
297 bidders (instead of number of bidders squared).

298 **[Insert Table 2 here]**

299 The results shows that the proposed Exponential model is superior to the Multinomial
300 model, generating less than half the (sum and mean) absolute estimation errors. Given that the
301 Exponential model is mathematically significantly simpler, this improvement is remarkable.

302

303 **Distribution of the number of new bidders**

304 So far we have implicitly made the assumption that N_i^* is a deterministic variable
305 whose average proportion or quantity can be approximated respectively by the Exponential
306 regression line described in equations (10) and (11). However, it is clear that given any number
307 of participating bidders $N_i > 0$, N_i^* can also take on different values. Particularly, $N_i^* = 0, 1, 2, \dots$
308 N_i .

309 Anticipating the distribution of possible N_i^* outcomes is equally important. First, to
310 anticipate how likely it is that each outcome happens. Second, because future bidding models
311 that incorporate randomly generated artificial bids will require a clear set of rules for modelling
312 different number of new bidders.

313 Further analysis from the simulation results from the urn model reveals that, for any
314 given value of N_i and i , the (unconditional) probability of $N_i^* = 0, 1, 2, \dots, N_i$ follows a Binomial
315 distribution with a number of trials $n = N_i$ and probability of success $p = N_i^*/N_i$, obtained from
316 expression (10). That is:

317
$$\text{Distribution}(N_i^*) = \text{Binomial}\left(n = N_i, p = \frac{N_i^*}{N_i}\right) \quad (13)$$

318 Proof of this can be found in the *Supplemental Online Material* file (“Binomial fit” tab),
319 which provides a number of examples of the simulations used to create Figure 1.

320 In the urn analogy, however, we assumed that N_i is constant throughout all auctions.
321 This is not usually the case in real auctions, where this number tends to vary substantially and,
322 most of the time, is difficult to anticipate. Hence, it remains necessary to check whether a
323 Binomial distribution can simulate closely enough the number of new bidders in real auctions.
324 Doing this involves converting expression (13) to a sum of Binomials, where each Binomial
325 contributes in the same proportion as auctions with different values of N_i appear in the dataset:

$$326 \quad \text{Distribution}(N_{i+1}^*) = \sum_{j=1}^{N_i} \left\{ \text{Freq}_j \cdot \text{Binomial} \left(n = j, p = \frac{N_i^*}{N_i} \right) \right\} \quad (14)$$

327 Freq_j represents then the proportion of auctions with $j=N_i$ bidders in the tender dataset. By
328 definition, the sum of all Freq_j values from 1 to N_i must equal 1.

329 The application of expression (14) to dataset 1 is shown in Figure 2. Shown are the
330 distribution fits for different auction group sizes. The figure shows that the average number of
331 new bidders (N_{i+1}^*) tends to decrease over time (as we analyze more auctions). Although not
332 reported, the results for datasets 2 and 3 are similar to those of dataset 1 (see *Supplemental*
333 *Online Material* file for results of those datasets).

334 **[Insert Figure 2 here]**

335 It is worth noting that the curves in Figure 2 are the result of a multitude of relatively
336 heterogeneous auctions. Each auction has a different number of participating bidders, N_i , and
337 population of bidders, N , that is constantly growing (see later) which affects the number of new
338 bidders, N_i^* , per auction. This causes both the number of trials, n , and the probability of
339 success, p , to change in each of the Binomial distributions (as per expression (14)).
340 Nevertheless, the visual appearance of the goodness of fit of the binomial expressions is

341 excellent. Similarly, all things considered, the p -values provided also seem to suggest a
342 satisfactory fit.

343 Finally, when introducing equation (13), we mentioned that it corresponds to the
344 *unconditional* probability of finding a given number of new bidders in the next auction. This
345 means that that expression is valid when the actual numbers of new bidders observed in
346 previous auctions are considered in average (expected) terms. In general, this is a necessary
347 assumption, as the population size of all potential bidders (N) is not known. However, if N was
348 indeed known it would be possible to resort to a more accurate model than the one offered by
349 the Binomial. That model corresponds to the Hypergeometric distribution, which actually fully
350 represents the urn analogy, presented earlier. Because it is rare for N to be known (or to be
351 accurately estimated) before the auction takes place, that model has been relegated to the
352 Appendix.

353

354 **The population of bidders**

355 As described earlier in expression (12), the population size of all potential bidders (N)
356 can be approximated as a function of the average values of a and N_i (a_{avg} and N_{avg}). Estimates
357 of N have many applications, probably the most important estimating the market size of
358 different construction sectors.

359 Calculating the value of N is mathematically very simple with the Exponential model.
360 When describing the urn analogy, we assumed that the population of bidders remained
361 constant. That is how we obtained the straight lines presented in Figure 1 (in log scale).
362 However, this is not usually the case in real auctions, where the identity of participating bidders
363 changes over time, and the total number of participating bidders either increases, remains
364 stable, or decreases as auctions are completed.

365 Monitoring potential variations in N in the proposed model is straightforward. All that
366 is necessary is to plot the values of the N estimates by applying expression (12) after the
367 completion of each auction i . By observing the trend line, it is possible to infer whether the
368 population of bidders is shrinking, remaining stable, or increasing. Another sign of a significant
369 change in the population size is the coefficient a in expression (3) or a_{avg} in expressions (10)
370 and (12) changing across auctions. These variations can also be plotted, but its application has
371 no physical meaning beyond serving as a proxy of N changes.

372 Figure 3 plots the estimates of a and N over time for the three datasets. It shows that
373 for all three datasets, the population of potential bidders has been growing over time. A few
374 regression lines fitting the values of N as a function of i are also included for illustrative
375 purposes to better visualize the major trends in successive N estimates.

376 **[Insert Figure 3 here]**

377 Of particular interest is the plot of N in dataset 1. A similar regression line was also
378 shown by Ballesteros-Pérez & Skitmore (2016) for their Multinomial model (Figure 2 in their
379 paper). Whilst their regression model produced a similar curve ($y=N=38.477 \cdot x^{0.396}$) as ours,
380 their estimate of N had a high variation (i.e. $R^2=0.5611$). Fortunately, the proposed Exponential
381 model is capable of producing substantially more accurate estimates of N using a significantly
382 smaller database (number of auctions) and with higher coefficients of determination (i.e.
383 $R^2=0.9639$).

384

385 **Discussion**

386 Since Friedman (1956) and Gates' (1967) seminal work on Bid Tender Forecasting,
387 many other forecasting models have followed. Most of these try to anticipate the probability of
388 several bidding-related outcomes (the number of participating bidders, lowest and/or average

389 bids submitted, the presence of abnormally high or low bids, etc.), all with the intention of
390 either gaining a competitive edge (from the contractor's perspective) or implementing better
391 awarding criteria (from the contracting authority's perspective).

392 The construction industry is currently becoming both more competitive and more
393 specialized. Additionally, there is increasingly easier, quicker, cheaper, and more transparent
394 access to all kinds of information. Bidding information is the same. For example, the United
395 States (with the data.gov website), the UK (with data.gov.uk), and the European Union (with
396 the European Public Sector Information Platform), are examples of entities that have recently
397 launched initiatives to make non-personal government data available as open data. Each of
398 these platforms provides access to tens of thousands of governments-related datasets. The
399 procurement and bidding information of local, regional, and national contracting authorities
400 constitute a significant proportion of these datasets. Moreover, these datasets are constantly
401 growing and periodically updated.

402 In this context, Bid Tender Forecasting Models (BTFMs) are very likely to thrive and
403 become essential tools for enhancing construction projects and services procurement. BTFMs
404 work with historical information to make predictions about the future. Companies and
405 governments that take advantage of this increasingly massive amount of information will be
406 able to make much better decisions. For construction contractors this might mean making more
407 profits by being awarded more contracts and/or anticipating the contracts for which the
408 competition may be less intense. For contracting authorities, this might mean fine-tuning the
409 contract awarding criteria and allow a higher discrimination power over a population of
410 potential bidders whose size and composition can be monitored.

411 These are just a few examples of inferences that BTFMs implementing the variables
412 analyzed in this study will allow. Mathematical expressions have been provided for each of
413 those variables. Further contexts, implications, and limitations are also discussed here.

414 In particular, an urn model is proposed to model the number of new bidders in an
415 upcoming auction, N_{i+1}^* . Despite assuming a series of relatively stable conditions – a constant
416 number of bidders per auction, N_i , and a constant population of potential bidders, N – the model
417 is empirically accurate at anticipating N_{i+1}^* both when N_{i+1} is known and unknown.

418 Anticipating the total number of participating bidders in upcoming auctions (N_{i+1}) to
419 make better estimates of N_{i+1}^* , though, is not always possible. As noted in the literature review
420 section, most models for anticipating N_{i+1} have not gained significant improvements over the
421 pure random case, even when contract sizes of future auctions are known in advance. However,
422 it is relatively common, from past records or just because bidders regularly meet each other, to
423 know the number of future competitors with varying degrees of certainty. There are also
424 situations when the number of bidders is certain. This happens, for instance, when the owner
425 shortlists a specified number of bidders from some prequalification stage.

426 Finally, there are situations when either the maximum or minimum number of bidders
427 is known. This usually happens, respectively, when the total number of invitations extended
428 by the owner is known (although not all bidders may submit a bid), or when the owner states
429 that unless there is a minimum level of competition (a minimum number of bids received), the
430 contract will not be awarded. Therefore, both N_{i+1} known and unknown cases are worth
431 considering and predictions may require complementary information from both cases
432 sometimes.

433 Finally, the advanced reader may think that, once a first estimate of the population of
434 potential bidders N is available, this variable can be used in turn to improve future N_{i+1}^*
435 estimates. This is, for example, a necessary assumption to implement the Hypergeometric
436 model presented in the Appendix. Indeed, there is apparently no reason why forecasting future
437 values of N should not be possible, as the growth of N in Figure 3 tends to be mostly relatively

438 smooth. However, addressing this problem this way involves additional complexities and
439 limitations.

440 Concerning the additional complexities, once a series of (past) N estimates is available,
441 it is necessary to find out how to forecast the value of N accurately in auction $i+1$. This may
442 need to be achieved in a number of ways as the N estimates generally experience some degree
443 of (local) volatility. Possible approaches may involve weighting more heavily the most recent
444 estimates of N , or just taking a linear regression estimate of the value of N at auction $i+1$.
445 Obviously, constant updates of the latest N estimates may be necessary after every new auction
446 has been completed.

447 There are two further limitations. First, a relatively large number of past auctions may
448 be required to obtain a relatively stable estimate of N (between 20 and 30 in the three auction
449 datasets analyzed). This could make it impractical, as it would be significantly more
450 information-greedy than the model proposed in this paper. Second, even if we tried to forecast
451 N_{i+1} * using N , it would still be necessary to resort to expressions (10) to (12), as these provide
452 the only means of updating the N estimates. Therefore, any future model that tries to take
453 advantage of N estimates will need to make use of the model presented here first.

454

455 **Conclusions**

456 We propose an Exponential model based on an urn analogy to predict the number of
457 new bidders that will participate in an upcoming auction. This, or alternatively predicting the
458 proportion of the new versus total number of bidders participating in the next auction, is of
459 significant value to a number of construction stakeholders, mostly to enhance competitiveness.

460 Tests on three construction auction datasets shows that the absolute deviation errors of
461 the proposed Exponential model are around 50% smaller than those produced by Ballesteros-

462 Pérez & Skitmore's (2016) Multinomial model – the only model with a similar aim found in
463 the literature. Moreover, the proposed model is mathematically simpler and has a much lower
464 computational cost. Indeed, the calculations could be carried out manually if necessary.
465 Furthermore, we show that the statistical distribution of new bidders closely resembles the sum
466 of a series of Binomial distributions. We also analyzed the variation (growth) of the population
467 of potential bidders by means of the same Exponential model. Finally, we briefly outlined a
468 number of applications of the model.

469 Regarding the model limitations, the exponential model heavily relies on a relatively
470 accurate estimate of the *total* number of participating bidders in the upcoming auction. That is
471 not a problem when we are only interested in the *proportion* of new versus total bidders, but it
472 is certainly limiting when we are interested in the *absolute* number of new bidders. Many
473 models in the past have attempted to come up with reliable estimates of the total number of
474 participating bidders and some examples have been reviewed in this paper. Besides the early
475 Poisson distribution model, many of them have resorted to multivariate regression. However,
476 most of these models have shown to be strongly context-specific (same country, economic
477 environment, client, type of project, etc.) and hardly provide reliable results in more generic
478 settings. Hence, future research may investigate multivariate approaches, particularly on trying
479 to accommodate the information of subsequent bids from previous bidders.

480

481 **Appendix**

482 The (conditional) probability of k new bidders participating in auction $i+1$ (i.e., N_{i+1}^*)
483 in the urn analogy actually follows a Hypergeometric distribution. The Hypergeometric
484 distribution is a discrete distribution that describes the probability of getting k successes
485 (drawing k objects with a particular feature) in n draws, *without replacement*, from a finite

486 population of size N . That population N contains exactly K objects with that feature (in this
 487 case that the bidder is new). Therefore, each draw is either a success or a failure.

488 Using the previous notation, the Probability Mass Function (PMF) of the
 489 Hypergeometric can be expressed as follows:

$$\begin{aligned}
 \text{Distribution}(k = N_{i+1}^*) &= \text{Hypergeometric} \left(\begin{array}{l} k = n^\circ \text{ of successes} = N_{i+1}^* \\ n = n^\circ \text{ of trials} = N_{i+1} \\ N = \text{population size} = N \\ K = n^\circ \text{ of objects} = N - N_1^* \end{array} \right) = \\
 490 \quad &= \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}} = \frac{\binom{N-N_1^*}{k} \binom{N_1^*}{N_{i+1}-k}}{\binom{N}{N_{i+1}}} = \quad (15) \\
 &= \frac{(N-N_1^*)!}{k!(N-N_1^*-k)!} \cdot \frac{(N_1^*)!}{(N_{i+1}-k)!(N_1^*-N_{i+1}+k)!} \cdot \frac{N_{i+1}!(N-N_{i+1})!}{N!}
 \end{aligned}$$

491 The problem when implementing expression (15) is, obviously, that N is generally not
 492 known, what is more, it may be changing over time. If we try to infer N from the random
 493 observations (number of new bidders from past auctions) we face another problem. The mean
 494 of the Hypergeometric distribution corresponds to $n \cdot (K/N)$. Note that this mean coincides with
 495 the mean of the Binomial distribution, which is $n \cdot p$. Thus, as from expression (13) we can infer
 496 that

$$497 \quad np = n \frac{N_{i+1}^*}{N_{i+1}} = n \frac{N - N_1^*}{N} = n \frac{K}{N} \quad (16)$$

498 Therefore, when implementing the Binomial model suggested earlier when trying to
 499 infer the probabilities of finding a given number of new bidders in the next auction we are
 500 already using the best estimates we have available. These estimates will be very accurate as
 501 long as $N_i \ll N$, which is generally the case in real contexts.

502 However, the Hypergeometric model also offers a new way of estimating N after each
503 auction has occurred. Working with expression (16) we can infer that once auction i has been
504 completed, the best estimate of N corresponds to the one that fulfils the following equality:

$$505 \quad N_i \frac{N - N_1^{i-1} *}{N} = N_i * \quad (17)$$

506 Therefore, by obtaining the variable N using expression (17) we can infer that the best
507 estimate of N after auction i corresponds to:

$$508 \quad N = \frac{N_i \cdot N_1^{i-1} *}{N_i - N_i *} \quad (18)$$

509 The problem with this expression is that it suffers from high volatility. It is almost
510 identical to that proposed by Ballesteros-Pérez & Skitmore (2016), which has proven to be
511 more disadvantageous than the ones proposed in expressions (7) and (12). Therefore, as a
512 general rule, the latter are preferred over expression (18).

513

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517 Gobierno de España (Spain).

518

519 **Data Availability**

520 The data generated or analyzed during the study are available from the corresponding
521 author by request.

522

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- 644

<i>Dataset</i>	<i>Source</i>	<i>Description</i>	<i>Period</i>	<i>N° bids</i>	<i>N° contracts</i>	<i>N_{avg}</i>
1	Skitmore (1986)	London building contracts	1976-77	1,915	373	5.13
2	Skitmore (1986)	North of England public works contracts	1979-82	1,235	218	5.67
3	Fu (2004)	Hong Kong Administrative Services Dept. contracts	1991-96	3,445	266	13.30

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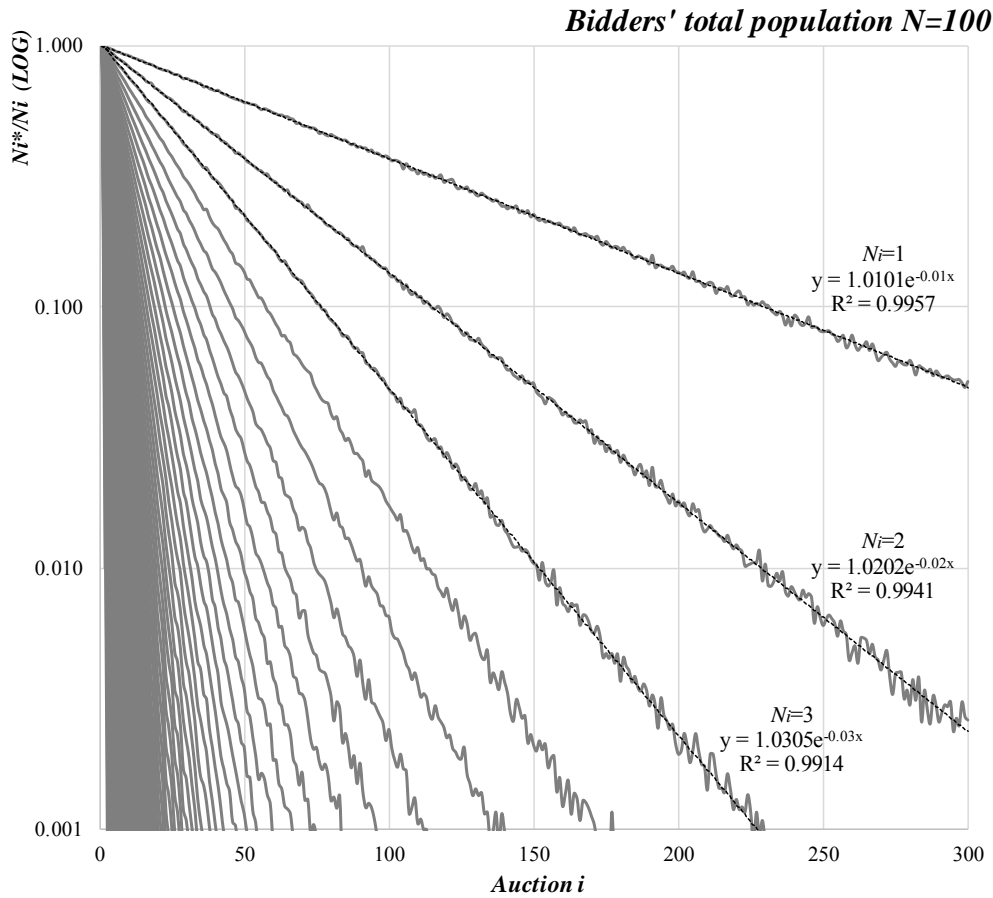
Table 1: Descriptive summary of the datasets of construction tenders

<i>Dataset</i>	<i>Absolute deviations</i>	<i>N_{i+1} known</i>		<i>N_{i+1} unknown</i>	
		<i>Exponential</i>	<i>Multinomial</i>	<i>Exponential</i>	<i>Multinomial</i>
1	<i>Sum</i>	50.53	103.22	280.96	497.50
	<i>Average</i>	0.14	0.28	0.76	1.33
	<i>Maximum</i>	1.00	1.28	4.36	5.85
2	<i>Sum</i>	25.64	32.39	144.25	274.32
	<i>Average</i>	0.12	0.15	0.67	1.26
	<i>Maximum</i>	0.74	0.90	5.09	5.11
3	<i>Sum</i>	8.63	10.15	124.94	144.54
	<i>Average</i>	0.03	0.05	0.47	0.65
	<i>Maximum</i>	0.67	0.77	8.87	10.17

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Table 2: Performance of the Exponential and Multinomial models.

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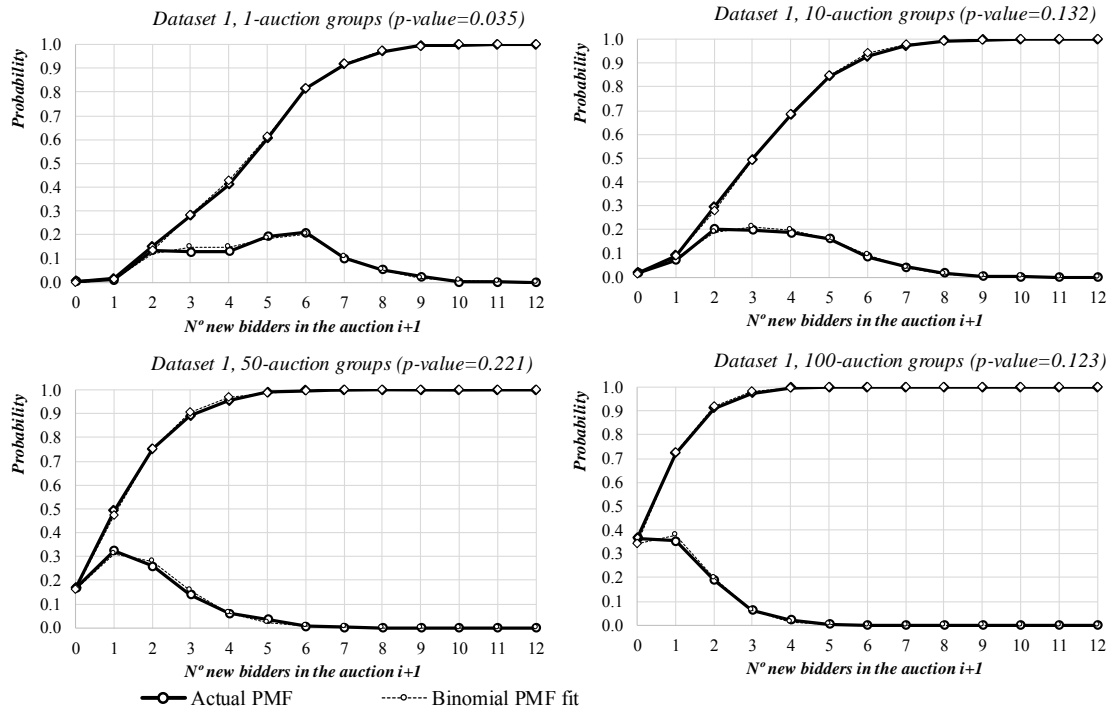


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649

Fig. 1. Urn analogy with $N=100$ bidders and N_i curves 1 to 100.

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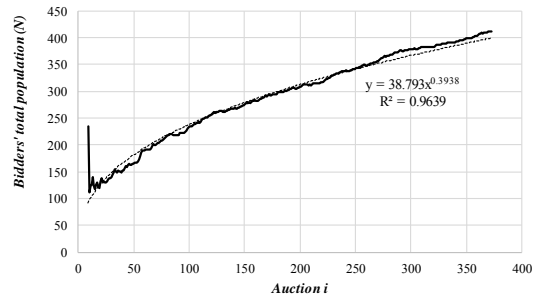
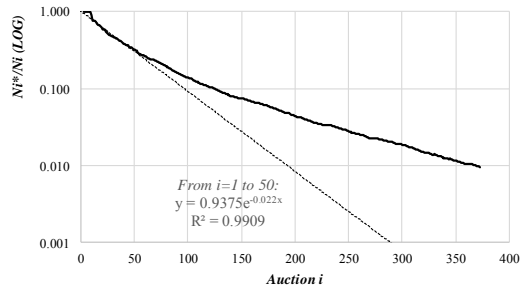
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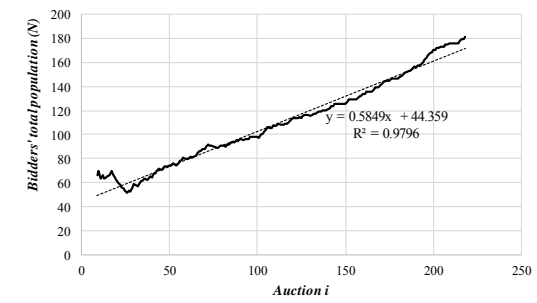
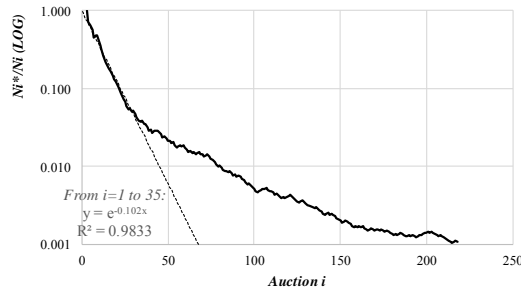
Fig. 2. Binomial distribution fit to the N_{i+1}^* values when auctions are analyzed in groups of

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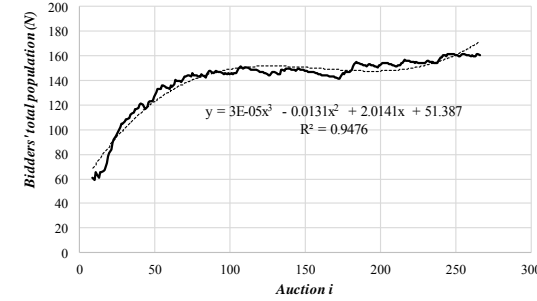
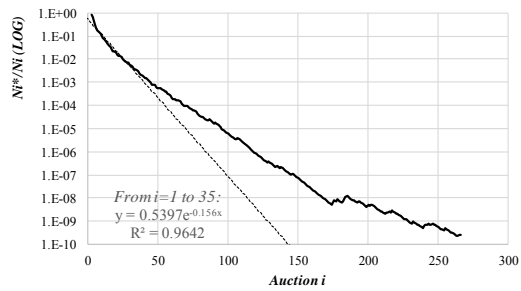
1, 10, 50 and 100.



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Fig. 3. Variation of a coefficients and population of bidders, N , for the three datasets of

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construction tenders.