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Rajaguru, Gulasekaran; Abeysinghe, Tilak

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Rajaguru Gulasekaran

THE DISTORTIONARY EFFECTS OF TEMPORAL AGGREGATION ON GRANGER CAUSALITY

Rajaguru GULASEKARAN
Department of Economics
National University of Singapore
10 Kent Ridge Crescent
Singapore 119260

email:
artp9449@nus.edu.sg

ABSTRACT

Economists often have to use temporally aggregated data in causality tests. A number of theoretical studies have pointed out that temporal aggregation has distorting effects on causal inference. This paper examines the issue in detail by plugging in theoretical cross covariances into the limiting values of least squares estimates. An extensive Monte Carlo study is conducted to examine small sample results. An empirical example is also provided. It is observed that in general the most distorting causal inferences are likely at low levels of aggregation where the order of aggregation just exceeds the actual causal lag. At high levels of aggregation, causal information concentrates in contemporaneous correlations. At present, a data-based approach is not available to establish the direction of causality between contemporaneously correlated variables.

1. INTRODUCTION

The use of highly temporally aggregated data for causal inference is quite common in the applied econometric literature. Some commonly investigated cases are the causality between economic growth and export growth, economic growth and trade, and economic growth and financial development. On one side are those who use Granger causality tests with mostly quarterly or annual data (see, for example, Jung and Marshall 1985, Rao 1989, Demitriades and Hussein 1996). On the other side are those who use cross-country regressions with data averaged over many years. Causality in these studies is pre-imposed and testing is done on the contemporaneous correlations (see, for example, Feder 1983, Kormendi and Merguire 1985, Ram 1986, Grier and Tullock 1989, Barro 1991, Levine and Renelt 1992, King and Levine 1993, Levine and Zervos 1993, Frankel and Roamer 1999). Both approaches suffer from the problems of temporal aggregation. The objective of this paper is to examine how temporal aggregation affects causal relationships among variables.

There is a sizable theoretical literature that investigates the impact of temporal aggregation on ARIMA models (see Wei, 1990, and references therein). A number of studies have also focused on temporal aggregation and the dynamic relationships between variables and shown that temporal aggregation weakens the distributed lag relationships (Telser 1967, Zellner and Montmarquette 1971, Sims 1971, Wei and Tiao 1975, Tiao and Wei 1976, Wei 1978, Wei and Metha 1980). Wei (1982), using Geweke's decomposition of a linear relationship, finds that temporal aggregation turns one-way causality into a feedback system. Campos et al. (1990) find that phase averaging in business cycle analysis produces inconsistent estimates and induces endogeneity into previously exogenous variables. Ericsson et al. (1994) examine how seasonal adjustment filters, which essentially embody a form of temporal aggregation, alter the short run dynamics while preserving cointegrating relationships. Ericsson et al. (2000) highlight the misspecifications involved in cross-country regressions that involve heavy temporal aggregations. Marcellino (1999) derives the vector ARIMA form of a temporally aggregated process (see also Lütkepohl, 1987) and shows that integration (unit roots) and cointegration are invariant to temporal aggregation, but many other aspects such as seasonal unit roots, exogeneity, causality, impulse responses, trend-cycle components, measures of persistence and forecasting are all affected by the aggregation process[1].

Although these studies have already pointed out some potential problems associated with temporally aggregated data, a comprehensive study that focuses on Granger causality alone would still be of immense value because of the practical significance of causality testing based on aggregated data. Our study looks into this problem in detail and provides some new insights.

In the next section, we derive the theoretical cross covariance between aggregated and disaggregated processes. This result plays a fundamental role in our exercise and is applicable to both stationary and integrated processes. In Section 3, we then derive the limiting values of least squares estimates of a VAR(1) process under different levels of temporal aggregation. In Section 4 we summarize the findings of an extensive Monte Carlo study. Section 5 provides a unique empirical example. In the concluding section we

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[1] Marcellino (1999) provides a long list of references (both theoretical and empirical) where these points have been previously established. On the empirical side Rossana and Seater (1992, 1995) find that the effects of temporal aggregation are much larger compared to cross-sectional aggregation.

summarize the results and highlight some important issues involved in Granger causality testing with temporally aggregated data.

2. RELATIONSHIP BETWEEN CROSS COVARIANCES OF DISAGGREGATE AND AGGREGATE SERIES

Let $z_t = (z_{1t}, z_{2t},\ldots, z_{nt})$ be a vector of basic disaggregate series and Z_t be the temporally aggregated vector. Temporal aggregation involves the construction of non-overlapping sums that can easily be obtained by defining the overlapping sum $X_t = (1 + L + ... + L^{m-1})z_t$ and then defining $Z_t = X_{mt}$. This is the same as systematic sampling of the X_t process at m intervals where m is a positive integer and is called the order of aggregation. For example, aggregating monthly data to quarterly figures involves setting m=3. Stram and Wei (1986) have derived the relationship between the autocovariances of the basic disaggregated series and the aggregated series for the univariate case. We extend their work to the multivariate case and examine how causal inferences are affected by the aggregation.

Let $w_t = (1-L)^d z_t$ be a weakly stationary process with mean zero and variance covariance matrix

$$\Gamma^w(k) = E(w_t w_{t-k}) = [\gamma_{ij}(k)], \quad i, j = 1,2,...,n \tag{1}$$

where $\gamma^w_{ii}(k)$ is the autocovariance of the i-th component, w_{it}, at lag k and $\gamma^w_{ij}(k)$ is the cross covariance between i-th and j-th components. Further $\gamma^w_{ii}(0)$ is the variance of the i-th series and $\gamma^w_{ij}(0)$ represents the contemporaneous cross covariance between the series.

Let L' be the backward shift operator on the aggregate time unit τ. Thus, $(1-L')Z_\tau = \quad Z_\tau - Z_{\tau-1} = X_{m\tau} - X_{m(\tau-1)} = (1-L^m)X_{m\tau}$. Let $W_\tau = (1-L')^d Z_\tau =$

$(1-L^m)^d X_{m\tau} = (1+L+.....+L^{m-1})^{d+1} w_{m\tau}$. Since W_τ is a finite moving average of a stationary process w_t, the d-th differenced aggregated series W_τ is also a covariance stationary process (Anderson, 1975). The cross covariance between $W_{i\tau}$ and $W_{j\tau\text{-}k}$ is given by

$$\gamma_{ij}^W(k) = Cov(W_{i\tau}, W_{j\tau-k}) = (1+L+L^2+......+L^{m-1})^{2(d+1)} \gamma_{ij}^w(mk+(d+1)(m-1)) \qquad (2)$$

where L operates on the index of $\gamma^w_{ij}(k)$ such that $L\gamma^w_{ij}(k) = \gamma^w_{ij}(k\text{-}1)$ (see Appendix for the derivation of 2). It may be useful to expresses (2) in matrix form as well:

$$\Gamma^W(k) = E(W_\tau W_{\tau-k}) = [\gamma_{ij}^W(k)],\ i,j = 1,2,...,n \qquad (3)$$

$$= (1+L+L^2+\cdots+L^{m-1})^{2(d+1)} \Gamma^w(mk+(d+1)(m-1))$$

where L operates on each element of the matrix $\Gamma^w(k)$. The basic relation given in (2) or (3) plays a crucial role in the assessment of the impact of temporal aggregation on Granger causality testing. Some special cases are discussed below.

3. CAUSAL INFERENCE FROM TEMPORALLY AGGREGATED DATA

To derive more specific results consider the following stationary bivariate VAR(1) system:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix},\ \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right). \qquad (4)$$

In this system the coefficients φ_{12} and φ_{21} measure the feedback between y_t and x_t, with $\varphi_{12} \neq 0$ implying Granger causality from x to y and $\varphi_{21} \neq 0$ implying Granger causality from y to x. We have set the contemporaneous correlation between the two error series to zero (i.e., $\rho_{12} = \sigma_{12} / \sigma_1 \sigma_2 = 0$) in order to assess the impact of temporal aggregation on this correlation.

The variances, autocovariances and cross-covariances of system (4) are given by

$$\gamma_{11}^{w}(0) = \sigma_y^2 = E(y_t y_t) = \varphi_{11}^2 \sigma_y^2 + \varphi_{12}^2 \sigma_x^2 + 2\varphi_{11}\varphi_{12}\gamma_{12}^{w}(0) + \sigma_1^2 \qquad (5)$$

$$\gamma_{22}^{w}(0) = \sigma_x^2 = E(x_t x_t) = \varphi_{21}^2 \sigma_y^2 + \varphi_{22}^2 \sigma_x^2 + 2\varphi_{21}\varphi_{22}\gamma_{12}^{w}(0) + \sigma_2^2 \qquad (6)$$

$$\gamma_{12}^{w}(0) = \gamma_{21}^{w}(0) = E(y_t x_t) = \varphi_{11}\varphi_{21}\sigma_y^2 + \varphi_{12}\varphi_{22}\sigma_x^2 + (\varphi_{11}\varphi_{22} + \varphi_{12}\varphi_{22})\gamma_{12}^{w}(0) \qquad (7)$$

$$\gamma_{11}^{w}(k) = E(y_t y_{t-k}) = \varphi_{11}\gamma_{11}^{w}(k-1) + \varphi_{12}\gamma_{21}^{w}(k-1) \qquad (8)$$

$$\gamma_{22}^{w}(k) = E(x_t x_{t-k}) = \varphi_{21}\gamma_{12}^{w}(k-1) + \varphi_{22}\gamma_{22}^{w}(k-1) \qquad (9)$$

$$\gamma_{12}^{w}(k) = E(y_t x_{t-k}) = \varphi_{11}\gamma_{12}^{w}(k-1) + \varphi_{12}\gamma_{22}^{w}(k-1) \qquad (10)$$

$$\gamma_{21}^{w}(k) = E(x_t y_{t-k}) = \varphi_{21}\gamma_{11}^{w}(k-1) + \varphi_{22}\gamma_{21}^{w}(k-1) \qquad (11)$$

Solving (5)-(7), we get

$$\sigma_y^2 = \frac{c_3\left[\sigma_1^2(b_2c_3 - b_3c_2) - \sigma_2^2(b_1c_3 - b_3c_1)\right]}{[a_1c_3 - a_3c_1][b_2c_3 - b_3c_2] - [a_2c_3 - a_3c_2][b_1c_3 - b_3c_1]} \qquad (12)$$

$$\sigma_x^2 = \frac{c_3\left[\sigma_1^2(a_2c_3 - a_3c_2) - \sigma_2^2(a_1c_3 - a_3c_1)\right]}{[b_1c_3 - b_3c_1][a_2c_3 - a_3c_2] - [b_2c_3 - b_3c_2][a_1c_3 - a_3c_1]} \qquad (13)$$

$$\gamma_{12}^{w}(0) = \frac{-\left[a_3\sigma_y^2 + b_3\sigma_x^2\right]}{c_3} \qquad (14)$$

where $a_1 = 1 - \varphi_{11}^2$, $b_1 = -\varphi_{12}^2$, $c_1 = -2\varphi_{11}\varphi_{12}$, $a_2 = -\varphi_{21}^2$, $b_2 = 1 - \varphi_{22}^2$, $c_2 = -2\varphi_{21}\varphi_{22}$, $a_3 = -\varphi_{11}\varphi_{21}$, $b_3 = -\varphi_{12}\varphi_{22}$ and $c_3 = 1 - [\varphi_{11}\varphi_{22} + \varphi_{12}\varphi_{21}]$.

Let Y_τ and X_τ be the *m*-period non-overlapping aggregates of y_t and x_t respectively. We now consider estimating the following bivariate VAR(1) from the temporally aggregated series:

$$\begin{pmatrix} Y_\tau \\ X_\tau \end{pmatrix} = \begin{pmatrix} \varphi_{11}^* & \varphi_{12}^* \\ \varphi_{21}^* & \varphi_{22}^* \end{pmatrix} \begin{pmatrix} Y_{\tau-1} \\ X_{\tau-1} \end{pmatrix} + \begin{pmatrix} E_{1\tau} \\ E_{2\tau} \end{pmatrix} \tag{15}$$

where $E_{i\tau}$ (*i=1,2*) represent the error process of the aggregated model. The OLS estimates $\hat{\varphi}_{ij}^*$ and $p\lim \hat{\varphi}_{ij}^*$ are given by:

$$\hat{\varphi}_{11}^* = \frac{\left(\sum Y_\tau Y_{\tau-1}\right)\left(\sum X_{\tau-1}^2\right) - \left(\sum Y_\tau X_{\tau-1}\right)\left(\sum Y_{\tau-1} X_{\tau-1}\right)}{\left(\sum Y_{\tau-1}^2\right)\left(\sum X_{\tau-1}^2\right) - \left(\sum Y_{\tau-1} X_{\tau-1}\right)^2}$$

$$p\lim \hat{\varphi}_{11}^* = \frac{\gamma_{11}^W(1)\gamma_{22}^W(0) - \gamma_{12}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2}, \tag{16}$$

and similarly

$$p\lim \hat{\varphi}_{12}^* = \frac{\gamma_{12}^W(1)\gamma_{11}^W(0) - \gamma_{11}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2} \tag{17}$$

$$p\lim \hat{\varphi}_{21}^* = \frac{\gamma_{21}^W(1)\gamma_{22}^W(0) - \gamma_{22}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2} \tag{18}$$

$$p\lim \hat{\varphi}_{22}^* = \frac{\gamma_{22}^W(1)\gamma_{11}^W(0) - \gamma_{21}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2}. \tag{19}$$

Using (2) the above parameters of the aggregate process can be expressed in terms of the moments of the disaggregated process and these in turn can be expressed in terms of the parameters of the original process using (5)-(14). Here we consider *m*=3, 12, and 60 to correspond to aggregating monthly data to quarterly, annual, and five-year aggregates[2]. Although we consider only the stationary case (d=0), the distortionary effects that we talk about are equally valid for non-stationary cases. The basic findings for d=0 and d=1 are similar though the magnitudes of the parameters are different. Note that d=1 involves aggregating I(1) series and then taking differences to make them stationary. In the case of cointegrated processes the model may be formulated as an error correction model in I(0) space (see Section 5).

Case 1: No Granger Causality Between the Variables in the Disaggregated Form

In this case $\varphi_{12} = \varphi_{21} = 0$ and with $\sigma_{12} = 0$ the two series are uncorrelated. Therefore, from (10), (11) and (14) $\gamma_{ij}^{w}(k) = 0$ for all k and $i \neq j$ $(i, j = 1,2)$. Further from (2) we can see that $\gamma_{ij}^{W}(k) = 0$ for all k and $i \neq j$. Thus, if the cross-covariances between the disaggregated series are zero then the cross-covariances between the aggregated series will also be zero. And from (17) and (18) we can see that $\varphi_{12}^{*} = \varphi_{21}^{*} = 0$. Thus, if there is no Granger causality between the disaggregated series then the Granger causality between the aggregated series will also be absent. Unfortunately, as we shall see later, the converse may not be true.

Case 2: Causality Between the Disaggregated Series is One-Sided

Let $\varphi_{12} = 0$ such that x_t does not Granger cause y_t. Accordingly, from (5)–(14) we get

$$\gamma_{11}^{w}(0) = \sigma_{y}^{2} \text{ and } \gamma_{11}^{w}(k) = \varphi_{11}\gamma_{11}^{w}(k-1) = \varphi_{11}^{k}\sigma_{y}^{2} \qquad (20)$$

[2] Many cross-country studies use long-term averages like those over five years.

$$\gamma_{12}^{w}(0)=\frac{\varphi_{11}\varphi_{21}\sigma_y^2}{1-\varphi_{11}\varphi_{22}} \Rightarrow \gamma_{12}^{w}(k)=\varphi_{11}\gamma_{12}^{w}(k-1)=\varphi_{11}^{k}\left(\frac{\varphi_{11}\varphi_{21}\sigma_y^2}{1-\varphi_{11}\varphi_{22}}\right)\forall k>0 \tag{21}$$

$$\gamma_{22}^{w}(k)=\varphi_{21}\gamma_{12}^{w}(k-1)+\varphi_{22}\gamma_{22}^{w}(k-1)=\varphi_{21}\varphi_{11}^{k-1}\left(\frac{\varphi_{11}\varphi_{21}\sigma_y^2}{1-\varphi_{11}\varphi_{22}}\right)+\varphi_{22}\gamma_{22}^{w}(k-1) \tag{22}$$

$$\gamma_{21}^{w}(k)=\varphi_{21}\gamma_{11}^{w}(k-1)+\varphi_{22}\gamma_{21}^{w}(k-1)=\varphi_{11}^{k-1}\varphi_{21}\sigma_y^2+\varphi_{22}\gamma_{21}^{w}(k-1)\forall k>0 \tag{23}$$

Thus, (17) changes to

$$p\lim\hat{\varphi}_{12}^{*}=\frac{\varphi_{11}\left(1+\varphi_{11}+\varphi_{11}^{2}+\ldots\varphi_{11}^{m-1}\right)^{2}\sigma_y^2\left[\gamma_{11}^{W}(0)\left(\frac{\varphi_{11}\varphi_{21}}{1-\varphi_{11}\varphi_{22}}\right)-\gamma_{12}^{W}(0)\right]}{\gamma_{11}^{W}(0)\gamma_{22}^{W}(0)-\left(\gamma_{12}^{W}(0)\right)^{2}} \tag{24}$$

and $p\lim\hat{\varphi}_{21}^{*}$ remains unchanged as in (18).

It is clear from the above expressions (and (2)) that when $\varphi_{11}=0$, $\varphi_{12}^{*}=0$, suggesting that if the one-sided causality runs from a white noise series to a stationary series in the disaggregated form then temporal aggregation will not produce a spurious feedback relationship. Similar inference does not apply when $\varphi_{22}=0$. In the following calculations we set $\varphi_{22}=0.5$ in order to produce results in terms of 3-dimentional graphs.

Figures 1a-1c show the effect of temporal aggregation on $p\lim\hat{\varphi}_{12}^{*}$ over the parameter ranges ($-.85\le\varphi_{11}\le..85$) and ($-1<\varphi_{21}<1$). To make the reading easier Table 1 provides $p\lim\hat{\varphi}_{12}^{*}$ for selected values of φ_{11} and φ_{21}. What is immediately noticeable is that as *m* increases VAR(1) tends to become VAR(0). However, when φ_{11} reaches unity, we get a near cointegrated specification and as a result VAR(1) remains

VAR(1) as m increases[3]. The most important observation is the creation of a spurious feedback effect as shown by the non-zero values of $p\lim\hat{\varphi}^{*}_{12}$. Interestingly when both φ_{11} and φ_{21} are of the same sign the feedback effect created is negative and when they are of opposite signs this becomes positive. The magnitude of the spurious feedback is large for large positive φ_{11}. Since large positive φ_{11} is more likely in practice, spurious feedback is very likely with temporally aggregated data. For certain parameter combinations as m increases the feedback effect first increases and then decreases.

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Figure 1 and Table 1

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Case 3: Granger Causality Between the Disaggregated Series is Bi-Directional

In this case both φ_{12} and φ_{21} are non-zero. The required aggregated parameters $(\varphi^{*}_{12}, \varphi^{*}_{21})$ are given in (17) and (18). To make computations easier and also to be used in the next section, we set $\varphi_{11} = 0$ and $\varphi_{22} = 0$. Accordingly the results in (8)-(14) specialize into

$$\gamma^{w}_{12}(0) = 0 \tag{25}$$

$$\gamma^{w}_{11}(1) = 0 \text{ and } \gamma^{w}_{11}(k) = \varphi_{12}\gamma^{w}_{21}(k-1) \tag{26}$$

$$\gamma^{w}_{22}(1) = 0 \text{ and } \gamma^{w}_{21}(k) = \varphi_{21}\gamma^{w}_{12}(k-1) \tag{27}$$

$$\gamma^{w}_{12}(1) = \varphi_{12}\sigma^{2}_{x} \text{ and } \gamma^{w}_{12}(k) = \varphi_{12}\gamma^{w}_{22}(k-1) \tag{28}$$

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[3] Because of this, in Figures 1a-1c we have restricted the range of φ_{11} to lie between -.85 and +.85 in order to highlight the shrinkage of VAR(1) to VAR(0).

$$\gamma_{21}^{w}(1)=\varphi_{21}\sigma_y^2 \text{ and } \gamma_{21}^{w}(k)=\varphi_{21}\gamma_{11}^{w}(k-1) \tag{29}$$

Through recursive substitution, we also get

$$\gamma_{11}^{w}(2k-1)=0, \quad \gamma_{11}^{w}(2k)=(\varphi_{12}\varphi_{21})^k\sigma_y^2 \quad \forall k=1,2,.... \tag{30}$$

$$\gamma_{22}^{w}(2k-1)=0, \quad \gamma_{22}^{w}(2k)=(\varphi_{12}\varphi_{21})^k\sigma_x^2 \quad \forall k=1,2,.... \tag{31}$$

$$\gamma_{12}^{w}(2k-1)=\varphi_{12}(\varphi_{12}\varphi_{21})^{k-1}\sigma_x^2, \quad \gamma_{12}^{w}(2k)=0 \quad \forall k=1,2,.... \tag{32}$$

$$\gamma_{21}^{w}(2k-1)=\varphi_{21}(\varphi_{12}\varphi_{21})^{k-1}\sigma_y^2, \quad \gamma_{21}^{w}(2k)=0 \quad \forall k=1,2,.... \tag{33}$$

$$\text{and } \sigma_x^2=\frac{1+\varphi_{21}^2}{1-\varphi_{12}^2\varphi_{21}^2}, \quad \sigma_y^2=\frac{1+\varphi_{12}^2}{1-\varphi_{12}^2\varphi_{21}^2} \tag{34}$$

For different values of *m* the expressions for $(\varphi_{12}^*, \varphi_{21}^*)$ can be evaluated using (30) – (34). Figures 2a-2c plot $p\lim\hat{\varphi}_{12}^*$ over the range $-1<\varphi_{12},\varphi_{21}<1$ and Table 2 provides a summary. As in the one-way causal system above, even in the feedback case the VAR(1) tends to become VAR(0) as *m* increases. What is more disturbing though is that a positive φ_{12} may become negative φ_{12}^*. Furthermore, the magnitudes of $p\lim\hat{\varphi}_{12}^*$ are such that in practice it is quite possible to conclude that causality is one-way though it is bi-directional.

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Figure 2 and Table 2

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Case 4: Contemporaneous Correlation

An important well-known problem of temporal aggregation is the creation of contemporaneous correlation even when such a correlation is absent. Using the VAR(1) system in (4) with $\varphi_{11} = 0$ and $\varphi_{22} = 0$ Ericsson et al. (2000) examined the effect of temporal aggregation on contemporaneous regression coefficient for *m*=2 and observed that this coefficient could be positive, negative, or zero. Here we generalize their result for any *m*. Note that with $\varphi_{11} = \varphi_{22} = \sigma_{12} = 0$ the contemporaneous correlation between y_t and x_t is zero (i.e., $\gamma_{12}^{w}(0) = 0$).

From the contemporaneous regression relationship $Y_t = cX_t + u_t$ with aggregated data we get

$$\hat{c} = \frac{\sum Y_\tau X_\tau}{\sum X_\tau^2}, \qquad \text{and } p\lim\ \hat{c} = \frac{\gamma_{12}^{W}(0)}{\gamma_{11}^{W}(0)} \quad . \tag{35}$$

As before (35) can be evaluated by substituting the relevant expressions for the disaggregated series from the previous results. Figures 3a-3c and Table 3 show results of $p\lim \hat{c}$ for different values of φ_{12} and φ_{21}. Here we let $\sigma_1^2 = \sigma_2^2 = 1$ for the ease of computation.

As observed by Ericsson et al. (2000) for *m*=2, the contemporaneous regression coefficient (also the correlation) could take positive, negative or zero at any level of aggregation. If both φ_{12} and φ_{21} are positive (negative) then the contemporaneous correlation will also be positive (negative). However, when the above parameters are of opposite signs then the sign of the contemporaneous correlation is determined by the sign of the larger of the two in absolute value.

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Figure 3 and Table 3

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4. MONTE CARLO RESULTS

The theoretical results presented above show how temporal aggregation creates spurious causal relations and Tables 1-3 show how the magnitude of the coefficients are affected asymptotically. It would be of interest to see how in small samples the standard test statistics such as *t* test would detect whether a coefficient is zero or not. To examine this we conducted a Monte Carlo study based on the VAR(1) process in (4) with $N(0, I)$ errors and recorded the rejection frequencies for $\varphi_{12} = 0$ based on the standard OLS based *t*-test. We also tested the hypothesis $\rho_{12} = 0$ based on the Breusch-Pagan Lagrange multiplier test, $\lambda_{LM} = T^* r_{12}^2 \sim \chi^2(1)$ where $T^* = T/m$ is the effective sample size and r_{12} is the correlation coefficient between the residuals of the two equations of (15) (see Lütkepohl, 1991).

Tables 4 and 5 provide some summary results that may be compared with Tables 1 and 2[4]. In general, we find that the temporal aggregation of causally unrelated series does not create any spurious causality at any level of aggregation. Concurring with the previous theoretical results, a common finding across all experiments is that as *m* increases VAR(1) becomes VAR(0) and lagged causality turns to instantaneous or contemporaneous causality. (The rejection frequency for $\rho_{12} = 0$ turns 100% when φ_{ij}

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[4] The results of a more extensive Monte Carlo study that cover all the three cases in Section 3 based on 500 replications, T=480 and m=1, 3, 6, 12, 24 are available in Gulasekaran (1999). The fall in the effective sample size as *m* increases is what we observe in practice. However, we also carried out a limited number of experiments by fixing the effective sample size at 160 and observed that the basic findings remain unaffected.

values become large). Absence of Granger-causality between highly aggregated series, therefore, does not necessarily mean that the disaggregated series are non-causal. Unfortunately, given that most data are available only in temporally aggregated form, the previous theoretical result that unrelated series remain unrelated after aggregation is of little use in practice for Granger causality testing because of the concentration of causal information in contemporaneous correlations.

The causality distortions discussed in the previous section are further highlighted in Tables 4 and 5. The rejection frequencies in Table 4 show that, in small samples, the conversion of one-way causality to a spurious feedback system becomes very prominent for small *m* and large values of φ_{11} and φ_{21}. Results in Table 5, on the other hand, show that when φ_{12} is small, at low levels of aggregations, a feedback system may be misdiagnosed as a one-way causal system[5].

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Table 4 and 5

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6. AN APPLICATION

In this section we present a unique empirical example to illustrate the distortionary effects of temporal aggregation on Granger causality. The example is unique because the main variable of the empirical model is available monthly in disaggregated form (which is very rear for economic time series) and the model resembles an ideal theoretical one. The example we consider is the following.

[5] Note that, though not strictly comparable, the rejection frequencies in Tables 4 and 5 represent the size and power of the test respectively.

To curb the car population, the Singapore government implemented a car quota system in August 1990. For this, cars were grouped into five categories according to their engine capacity (small, medium, large, luxury and open). To buy a new car the buyers first have to buy a piece of paper called the certificate of entitlement (COE). The price of the COE, known as the quota premium (QP), is decided through a monthly bidding process. The minimum successful bid within a quota becomes the quota premium. The monthly QP is not an aggregated series in any sense.

After considering a number of determinants of QP of various categories Lai (2001) finds that the only significant determinant of the QP of the luxury category is the performance of the stock market. He measures the latter by the "all equity price index" compiled by the Stock Exchange of Singapore (SES). In this section we examine the relationship between the QP of luxury cars and the above stock price index.[6] Note that the average monthly stock price index involves temporal aggregation. Here we ignore the dynamic relationship that exists between monthly QP and the disaggregated stock prices.

We denote the two variables by *y = ln(QP of luxury cars)* and *x = ln(Stock price index)*. For temporally aggregated data we take the average over *m* months of QP and stock price index separately and then take logarithms. Preliminary estimation shows that the most appropriate model for monthly data is a VAR(1) of the form (4) with $\varphi_{21} = 0$ and $\varphi_{22} = 1$. Moreover, the two error processes are also uncorrelated ($\sigma_{12} = 0$). This means that x_t is an exogenous random walk and Granger causality is unidirectional

[6] Our sample period is 1990M8-1999M4. The data since May 1999 are not usable because the government merged a number of car categories to form a different classification. The data on QP can be downloaded from the TREND database maintained by the Department of Statistics, Government of Singapore and the stock price data can be downloaded from the SES website.

from x to y. Johansen's contegration tests strongly suggest that the two variables are cointegrated. We, therefore, proceeded with the following VECM:

$$\begin{pmatrix} \Delta y_t \\ \Delta x_t \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}. \qquad (36)$$

Since x is exogenous we expect $\alpha_2 = 0$. Table 6 reports the estimation results for m=1 (no aggregation) to m=6. In the table we report the normalized cointegrating coefficient $\beta = -\beta_2 / \beta_1$. The estimates are based on the Johansen procedure in PCGIVE.

It should be noted that (36), being a cointegrated VAR(1) process, does not reduce to VAR(0) as m increases. As a result the contemporaneous cross correlation of the residuals (r_{12}) does not increase with m either. The results show that $\hat{\beta}$ remains roughly the same as m increases. This shows the invariance of cointegration we mentioned earlier. However, the magnitude of $\hat{\alpha}_1$ increases steadily and remains highly significant. The magnitude of $\hat{\alpha}_2$ also tends to increase though not in a systematic manner. Most importantly $\hat{\alpha}_2$ becomes significant at the 10% level for m=4 and m=6. As observed in our analytical results in Section 3 on spurious feedback, $\hat{\alpha}_2$ remains persistently negative. (Actually, since the same cointegrating vector enters both the equations we expect $\hat{\alpha}_1 < 0$ and $\hat{\alpha}_2 > 0$.) Despite the sharp drop in the effective sample size ($T^*=T/m$) when m increases, the example is highly instructive since it shows the possibility that temporal aggregation can create a spurious feedback to a strictly exogenous variable[7].

[7] Monte Carlo results in Mamingi (1996) based on a data generating process similar to (36) shows that the probability of detecting a spurious feedback increases dramatically when both T^* and m increase.

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Table 6

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5. CONCLUSION

Economists often have to use temporally aggregated or systematically sampled data in econometric models. Unfortunately many properties of the data generating process alter as a result of temporal aggregation and systematic sampling. In this paper we have presented a methodology to evaluate the magnitude of the Granger causality distortions resulting from temporal aggregation. While our results reaffirm previous theoretical findings we also find that most of the distortions occur only at low levels of aggregation where the order of aggregation just exceeds the true causal lag. At high levels of aggregations what is left would be only the contemporaneous correlation. The standard Granger causality tests that ignore the contemporaneous correlation have to be used with utmost care because a finding of "no causality" with temporally aggregated data does not necessarily mean "no causality" between the variables.

This means that the practitioner must have a good understanding about the causal lag. For example, the knowledge about how long it takes for the production and delivery to take place is important for a study on the relationship between orders and sales. The causal lag varies with the nature of the product and the data gathered must be as close as possible to the causal lag. Unfortunately, often, such data are not available. With temporally aggregated data a feedback system seems to be the norm. Although it makes a lot of sense to formulate an unrestricted VAR to account for the feedback, causality tests based on such models may have no correspondence to the underlying true causality. Given the significance of the contemporaneous correlation in temporally aggregated data, it does not make sense to throw away this information in the causality tests. Unfortunately only causal inference one could attach to contemporaneous correlation is that based on a priori information, a theory, a practice that economists have been following all along. This, however, takes us back to square one, the very dilemma the causality tests were trying to resolve.

One solution is to develop a causality test within a cointegration framework. Cointegration is invariant to temporal aggregation and implies Granger causality (Granger, 1988). Unfortunately at the moment there is no data-based approach to establish the direction of causality between two cointegrated variables. This is an area worth exploring.

Appendix 1: Derivation of (2)

Define the forward shift operator $F = L^{-1}$ such that $Fw_t = w_{t+1}$ and $F\gamma_{ij}(k) = \gamma_{ij}(k+1)$. Let c^i be the coefficient of L^i of the polynomial $(1+L+\ldots+L^{m-1})^{d+1}$.

$$\gamma_{ij}^{W}(k) = E[W_{it}W_{jt-k}]$$

$$= E[(1+L+\cdots+L^{m-1})^{d+1} w_{imt} (1+L+\cdots+L^{m-1})^{d+1} w_{jm(t-k)}]$$
$$= E[(c_0 w_{imt} + c_1 w_{imt-1} + \cdots c_{(d+1)(m-1)} w_{imt-(d+1)(m-1)})$$
$$(c_0 w_{jmt-mk} + c_1 w_{jmt-mk-1} + \cdots c_{(d+1)(m-1)} w_{jmt-mk-(d+1)(m-1)})$$

$$= c_0[c_0\gamma_{ij}^{w}(mk) + c_1\gamma_{ij}^{w}(mk+1) + \cdots + c_{(d+1)(m-1)}\gamma_{ij}^{w}(mk+(d+1)(m-1))]$$
$$+ c_1[c_0\gamma_{ij}^{w}(mk-1) + c_1\gamma_{ij}^{w}(mk) + \cdots + c_{(d+1)(m-1)}\gamma_{ij}^{w}(mk+(d+1)(m-1)-1)]$$
$$+ \cdots\cdots$$
$$+ c_{(d+1)(m-1)}[c_0\gamma_{ij}^{w}(mk-(d+1)(m-1)) + c_1\gamma_{ij}^{w}(mk-(d+1)(m-1)-1)\cdots + c_{(d+1)(m-1)}\gamma_{ij}^{w}(mk)]$$

$$= c_0[(1+F+\cdots F^{m-1})^{d+1}\gamma_{ij}^{w}(mk)] + c_1[(1+F+\cdots F^{m-1})^{d+1}\gamma_{ij}^{w}(mk-1)] + \cdots +$$
$$+ c_{(d+1)(m-1)}[(1+F+\cdots F^{m-1})^{d+1}\gamma_{ij}^{w}(mk-(d+1)(m-1)]$$

$$= (1+F+\cdots+F^{m-1})^{(d+1)}[c_0\gamma_{ij}^{w}(mk) + c_1\gamma_{ij}^{w}(mk-1) + \cdots + c_{(d+1)(m-1)}\gamma_{ij}^{w}(mk-(d+1)(m-1)]$$

$$= (1+F+\cdots+F^{m-1})^{(d+1)}(1+L+\cdots+L^{m-1})^{(d+1)}\gamma_{ij}^{w}(mk)$$

$$= F^{(d+1)(m-1)}(1+L+\cdots+L^{m-1})^{(d+1)}\gamma_{ij}^{w}(mk)$$

$$= (1+L+\cdots+L^{m-1})^{2(d+1)}\gamma_{ij}^{w}(mk+(d+1)(m-1)).$$

Thus,

$$\gamma_{ij}^{W}(k) = (1+L+\cdots+L^{m-1})^{2(d+1)}\gamma_{ij}^{w}(mk+(d+1)(m-1)).$$

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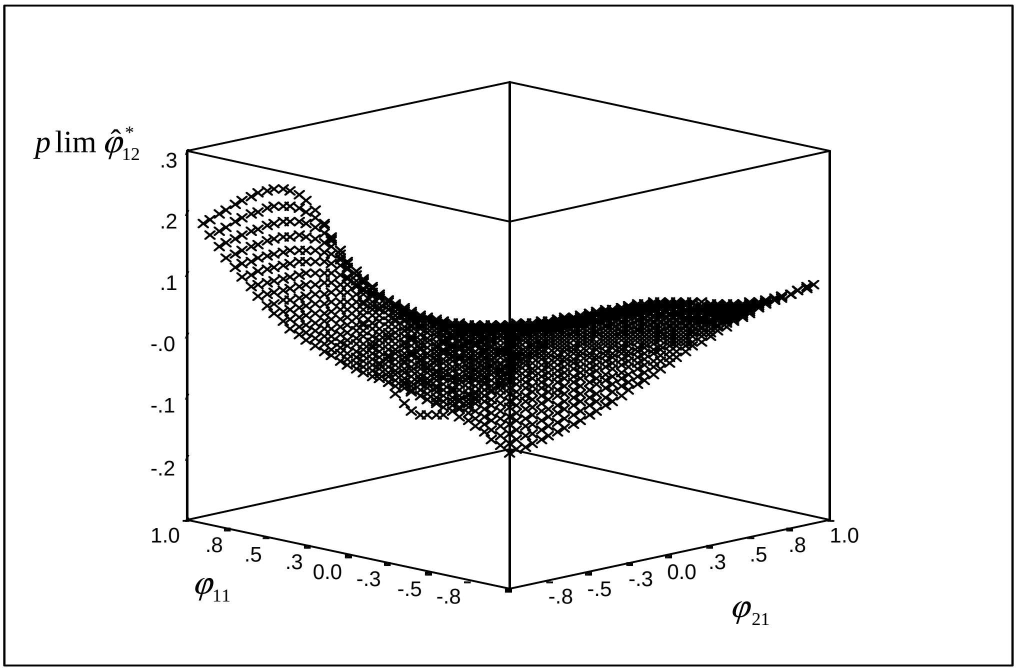

Figure 1a. Spurious feedback created by temporal aggregation, m=3

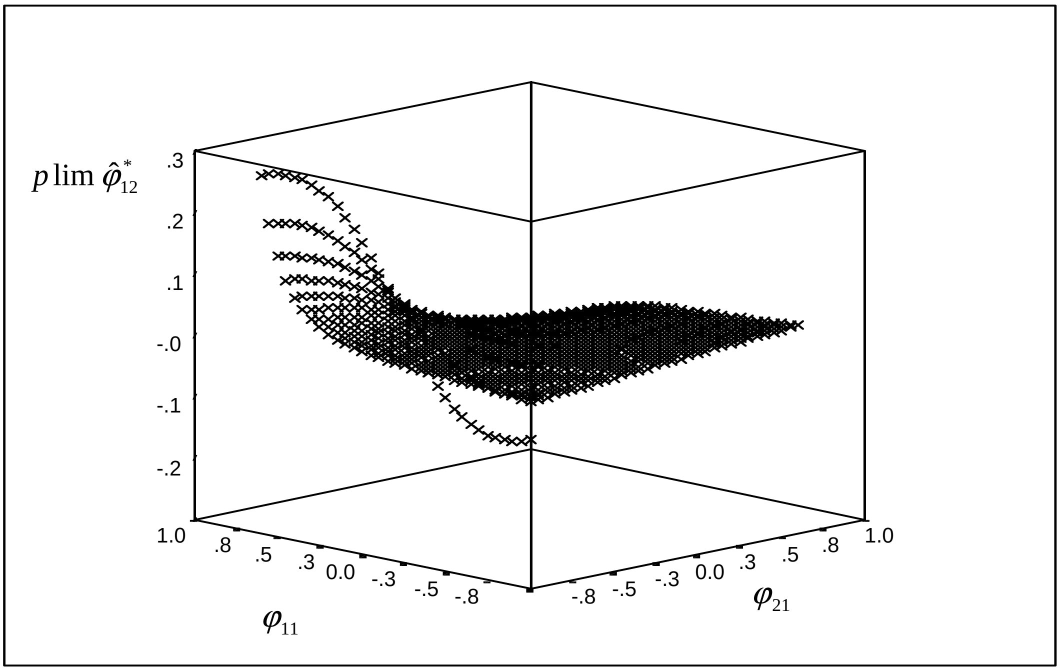

Figure 1b. Spurious feedback created by temporal aggregation, m=12

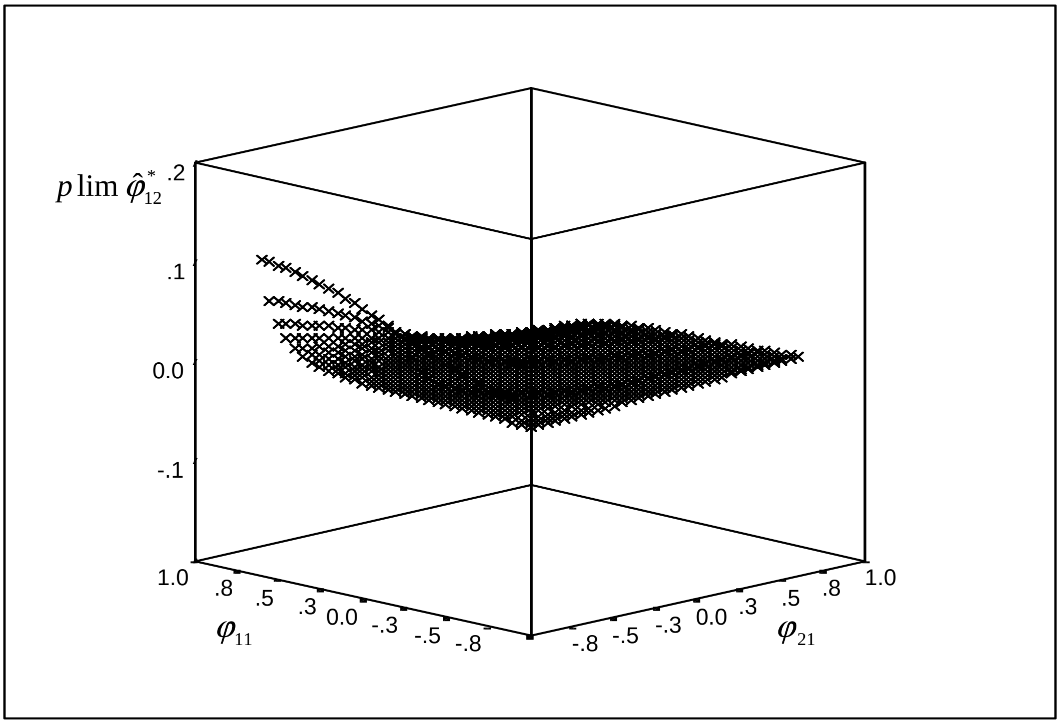

Figure 1c. Spurious feedback created by temporal aggregation, m=60

Table 1. Spurious feedback created by temporal aggregation:
Values of $p\lim\varphi_{12}^{*}$ when φ_{12} =0

φ_{21} across / φ_{11} down	**-0.95**	**-0.8**	**-0.5**	**-0.2**	**0**	**0.2**	**0.5**	**0.8**	**0.95**
m=3									
-0.95	-0.08	-0.08	-0.06	-0.02	0.00	0.02	0.06	0.08	0.08
-0.8	-0.06	-0.05	-0.04	-0.02	0.00	0.02	0.04	0.05	0.06
-0.5	-0.03	-0.03	-0.02	-0.01	0.00	0.01	0.02	0.03	0.03
-0.2	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.01
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.02	0.02	0.01	0.01	0.00	-0.01	-0.01	-0.02	-0.02
0.5	0.06	0.06	0.06	0.03	0.00	-0.03	-0.06	-0.06	-0.06
0.8	0.14	0.14	0.15	0.09	0.00	-0.09	-0.15	-0.14	-0.14
0.95	0.18	0.20	0.21	0.14	0.00	-0.14	-0.21	-0.20	-0.18
m=12									
-0.95	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.01
-0.8	-0.02	-0.02	-0.01	0.00	0.00	0.00	0.01	0.02	0.02
-0.5	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.01
-0.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.01	0.01	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01
0.5	0.05	0.04	0.03	0.01	0.00	-0.01	-0.03	-0.04	-0.05
0.8	0.26	0.26	0.23	0.12	0.00	-0.12	-0.23	-0.26	-0.26
0.95	0.75	0.80	0.83	0.52	0.00	-0.52	-0.83	-0.80	-0.75
m=60									
-0.95	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.02
-0.8	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01
-0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.01	0.01	0.01	0.00	0.00	0.00	-0.01	-0.01	0.00
0.8	0.12	0.10	0.07	0.03	0.00	-0.03	-0.07	-0.10	-0.12
0.95	1.24	1.24	1.06	0.53	0.00	-0.53	1.06	-1.24	-1.24

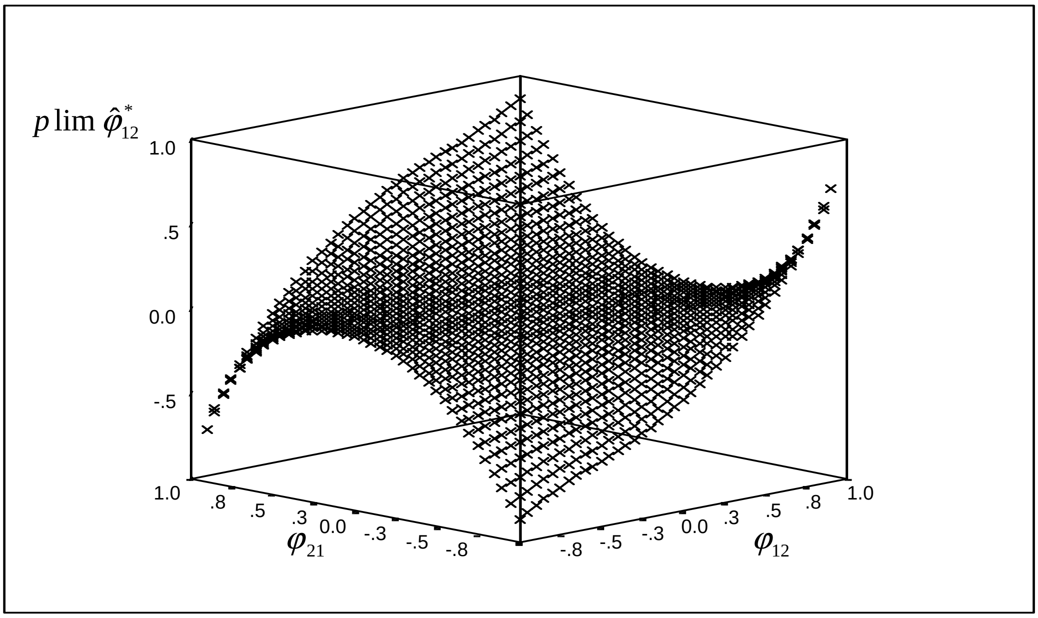

Figure 2a. $p\lim \hat{\varphi}^{*}_{12}$ from a feedback system, m=3

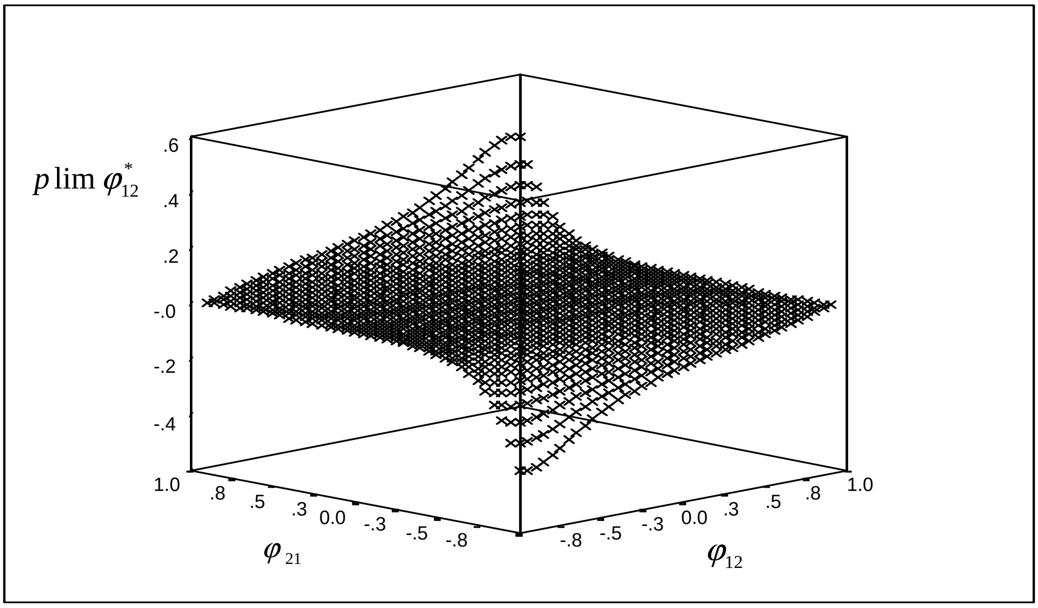

Figure 2b. $p\lim \hat{\varphi}^{*}_{12}$ from a feedback system, m=12

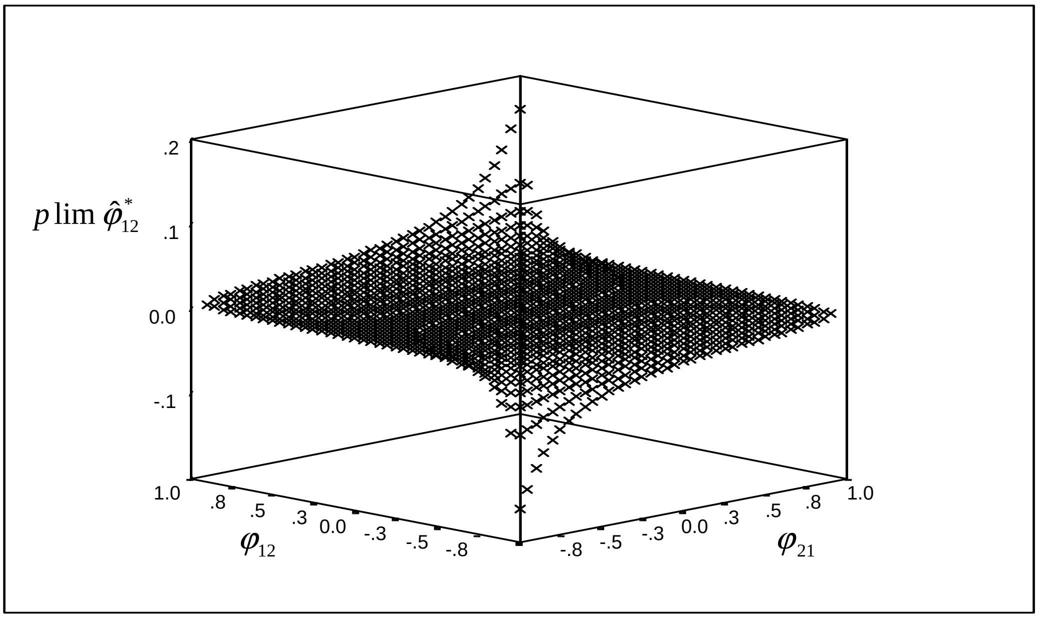

Figure 2c. $p\lim\hat{\varphi}^*_{12}$ from a feedback system, m=60

Table 2. Effects of temporal aggregation on a feedback system
Values of $p\lim\varphi_{12}^{*}$ when $\varphi_{12} \neq 0, \varphi_{21} \neq 0$

m=3									
φ_{21} across φ_{12} down	**-0.95**	**-0.8**	**-0.5**	**-0.2**	**0**	**0.2**	**0.5**	**0.8**	**0.95**
-0.95	-0.88	-0.79	-0.65	-0.52	-0.40	-0.25	0.05	0.44	0.71
-0.8	-0.67	-0.58	-0.49	-0.40	-0.32	-0.22	-0.01	0.24	0.39
-0.5	-0.26	-0.25	-0.23	-0.21	-0.18	-0.14	-0.06	0.03	0.08
-0.2	-0.06	-0.06	-0.07	-0.07	-0.07	-0.06	-0.04	-0.02	-0.01
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.01	0.02	0.04	0.06	0.07	0.07	0.07	0.06	0.06
0.5	-0.08	-0.03	0.06	0.14	0.18	0.21	0.23	0.25	0.26
0.8	-0.39	-0.24	0.01	0.22	0.32	0.40	0.49	0.58	0.64
0.95	-0.71	-0.44	-0.05	0.25	0.40	0.52	0.65	0.79	0.88
m=12									
-0.95	-0.39	-0.37	-0.25	-0.17	-0.13	-0.11	-0.07	-0.03	-0.01
-0.8	-0.16	-0.18	-0.15	-0.12	-0.10	-0.08	-0.06	-0.03	-0.01
-0.5	-0.03	-0.04	-0.06	-0.05	-0.05	-0.04	-0.03	-0.02	-0.01
-0.2	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.01	-0.01	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.00	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
0.5	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.04	0.03
0.8	0.01	0.03	0.06	0.08	0.10	0.12	0.15	0.18	0.16
0.95	0.01	0.03	0.07	0.11	0.13	0.17	0.25	0.37	0.39
m=60									
-0.95	-0.16	-0.10	-0.05	-0.04	-0.03	-0.02	-0.02	-0.01	-0.01
-0.8	-0.03	-0.04	-0.03	-0.03	-0.02	-0.02	-0.01	-0.01	0.00
-0.5	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00
-0.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00
0.8	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.03
0.95	0.01	0.01	0.02	0.02	0.03	0.04	0.05	0.10	0.16

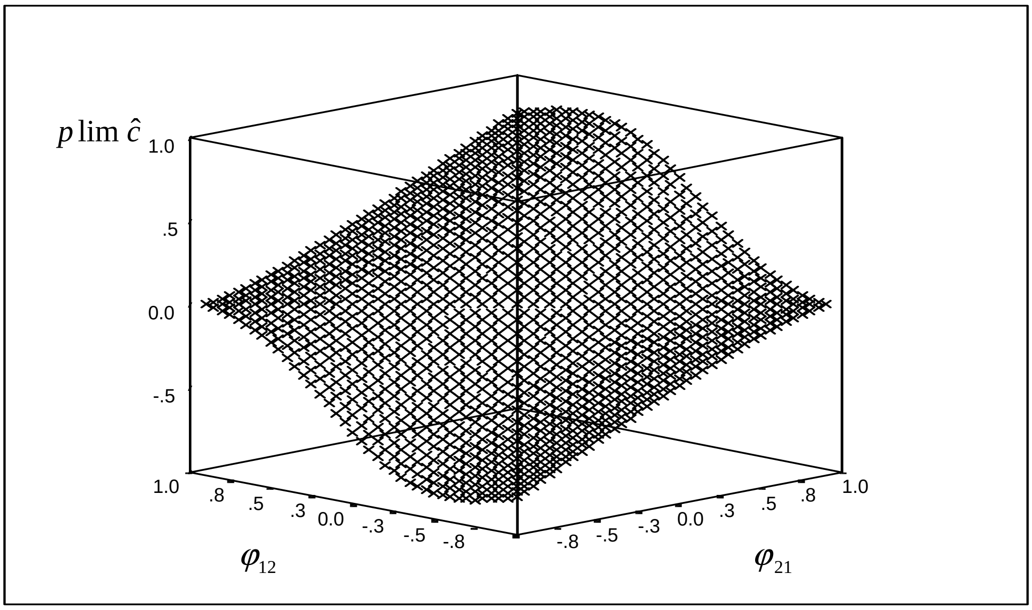

Figure 3a. Contemporaneous regression coefficient, m=3

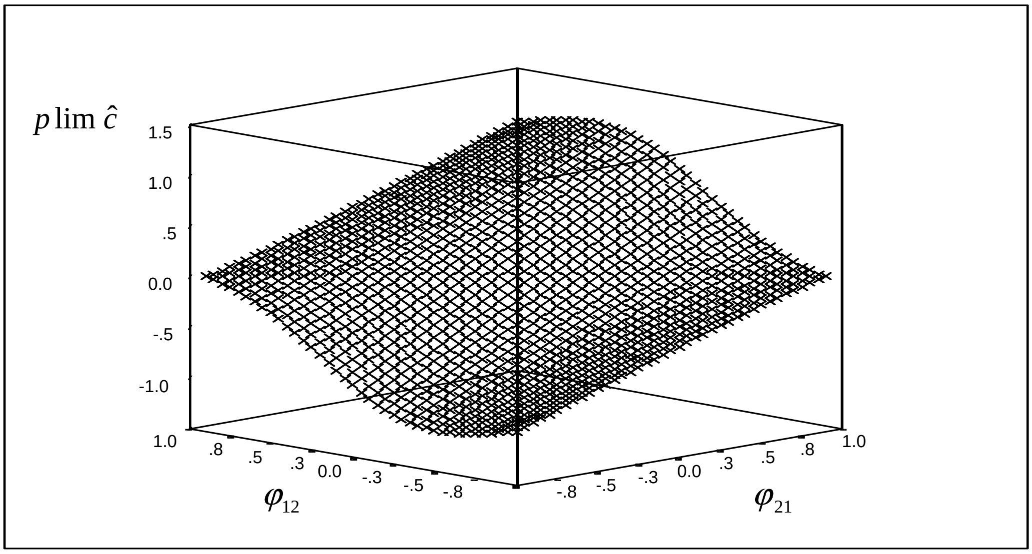

Figure 3b. Contemporaneous regression coefficient, m=12

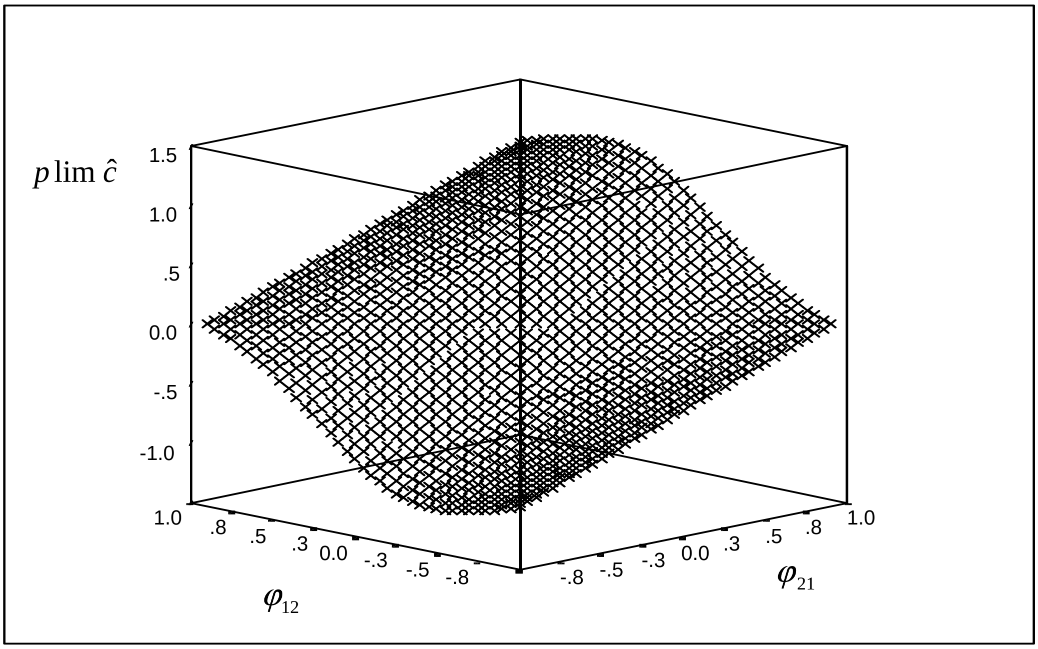

Figure 3c. Contemporaneous regression coefficient, m=60

Table 3. Contemporaneous regression coefficient $p\lim\hat{c}$

M=3									
φ_{21} across φ_{12} down	**-0.95**	**-0.8**	**-0.5**	**-0.2**	**0**	**0.2**	**0.5**	**0.8**	**0.95**
-0.95	-0.79	-0.72	-0.57	-0.43	-0.33	-0.24	-0.12	-0.03	0.00
-0.8	-0.83	-0.75	-0.58	-0.43	-0.33	-0.23	-0.10	0.00	0.03
-0.5	-0.87	-0.77	-0.57	-0.39	-0.27	-0.15	0.00	0.13	0.18
-0.2	-0.78	-0.67	-0.46	-0.26	-0.10	0.00	0.19	0.36	0.45
0	-0.63	-0.53	-0.33	-0.13	0.00	0.13	0.33	0.53	0.63
0.2	-0.45	-0.36	-0.19	0.00	-0.13	0.26	0.46	0.67	0.78
0.5	-0.18	-0.13	0.00	0.15	0.27	0.39	0.57	0.77	0.87
0.8	-0.03	0.00	0.10	0.23	0.33	0.43	0.58	0.75	0.83
0.95	0.00	0.03	0.12	0.24	0.33	0.43	0.57	0.72	0.79
m=12									
-0.95	-0.99	-0.90	-0.73	-0.57	-0.46	-0.35	-0.19	-0.05	0.00
-0.8	-1.05	-0.95	-0.76	-0.57	-0.45	-0.33	-0.15	0.00	0.06
-0.5	-1.12	-1.00	-0.76	-0.52	-0.37	-0.22	0.00	0.20	0.29
-0.2	-1.04	-0.90	-0.63	-0.35	-0.18	0.00	0.26	0.51	0.64
0	-0.87	-0.73	-0.46	-0.18	0.00	0.18	0.46	0.73	0.87
0.2	-0.64	-0.51	-0.26	0.00	0.18	0.35	0.63	0.90	1.04
0.5	-0.29	-0.20	0.00	0.22	0.37	0.52	0.76	1.00	1.12
0.8	-0.06	0.00	0.15	0.33	0.45	0.57	0.76	0.95	1.05
0.95	0.00	0.05	0.19	0.35	0.46	0.57	0.73	0.90	0.99
m=60									
-0.95	-1.00	-0.92	-0.76	-0.60	-0.49	-0.38	-0.23	-0.07	0.00
-0.8	-1.06	-0.97	-0.79	-0.60	-0.48	-0.36	-0.18	0.00	0.08
-0.5	-1.15	-1.03	-0.79	-0.55	-0.39	-0.24	0.00	0.23	0.34
-0.2	-1.09	-0.95	-0.66	-0.38	-0.19	0.00	0.28	0.56	0.70
0	-0.93	-0.79	-0.49	-0.20	0.00	0.20	0.49	0.79	0.93
0.2	-0.70	-0.56	-0.28	0.00	0.19	0.38	0.66	0.95	1.09
0.5	-0.34	-0.23	0.00	0.24	0.39	0.55	0.79	1.03	1.15
0.8	-0.08	0.00	0.18	0.36	0.48	0.60	0.79	0.97	1.06
0.95	0.00	0.07	0.23	0.38	0.49	0.60	0.79	0.92	1.00

Table 4. Rejection frequencies (%) for H_0: φ_{12}=0 when φ_{12}=0
(One-way causal system, φ_{22}=0.5, T=480, 2000 replications, $\alpha = 5\%$)

φ_{21} across / φ_{11} down	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
M=3									
0.1	5.0	4.1	5.3	5.3	4.8	5.5	5.8	4.8	4.9
0.2	4.6	5.8	4.5	4.8	4.6	6.3	5.0	6.0	6.2
0.3	4.4	4.3	5.8	6.0	7.4	6.8	8.4	9.1	9.5
0.4	4.6	5.4	6.4	7.8	8.5	10.6	12.4	12.5	15.5
0.5	5.8	5.9	8.2	10.7	12.0	16.0	19.4	21.2	21.9
0.6	5.5	8.0	11.6	16.6	19.9	24.2	27.7	31.2	33.9
0.7	7.1	9.6	15.2	23.3	31.4	36.6	40.8	47.6	49.4
0.8	7.8	13.2	24.1	34.9	45.0	52.3	58.8	62.7	67.1
0.9	8.4	18.2	32.2	46.9	59.5	70.1	76.1	79.6	81.2
M=12									
0.1	5.6	4.8	5.0	5.2	5.8	4.4	5.0	4.9	5.8
0.2	5.8	5.4	5.2	5.0	5.8	5.1	5.5	5.3	5.1
0.3	5.0	5.2	5.4	5.0	5.5	5.8	5.4	5.7	5.8
0.4	5.4	5.7	5.1	4.6	5.4	5.5	5.7	5.8	5.7
0.5	5.1	5.2	5.8	5.6	5.4	6.1	6.7	6.2	6.4
0.6	5.3	5.1	5.2	5.2	6.0	6.3	7.7	7.4	7.7
0.7	5.8	6.1	5.0	6.9	9.1	9.1	10.3	11.1	12.5
0.8	5.4	6.5	9.8	10.6	13.0	14.7	17.8	20.1	23.4
0.9	6.4	11.4	16.2	21.7	28.3	34.0	39.4	43.9	46.6

Table 5. Rejection frequencies (%) for H_0: $\varphi_{12}=0$ when $\varphi_{12}\neq0$

(Feedback system $\varphi_{12}\neq0$, $\varphi_{21}\neq0$, $\varphi_{11}=\varphi_{22}=0$, T=480, 2000 replications, $\alpha = 5\%$)

M=3									
φ_{21} across φ_{12} down	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	6.4	6.2	7.2	6.3	6.4	5.3	6.3	5.8	6.5
0.2	11.8	11.9	11.2	12.0	10.8	13.7	11.7	12.1	11.5
0.3	24.7	23.9	25.3	26.0	24.5	25.7	25.4	25.5	26.4
0.4	36.3	41.2	42.4	43.0	43.6	45.9	48.4	51.6	55.9
0.5	51.4	58.0	60.0	64.3	68.0	70.5	73.8	80.4	85.4
0.6	68.9	73.6	78.7	79.8	85.3	89.5	92.1	95.8	97.5
0.7	81.0	85.9	89.2	92.9	95.7	97.7	98.8	99.7	99.9
0.8	88.5	93.1	96.9	97.9	98.8	99.6	99.9	100.0	100.0
0.9	95.0	97.5	98.8	99.6	99.9	100.0	100.0	100.0	100.0
m=12									
0.1	5.1	5.3	6.0	4.5	5.0	4.8	5.3	4.6	5.9
0.2	5.8	5.7	5.7	4.9	5.5	5.2	5.0	5.6	4.3
0.3	5.4	5.3	5.4	5.6	5.5	5.8	5.6	4.9	5.4
0.4	5.4	6.3	6.1	5.8	5.9	5.9	4.2	5.7	5.4
0.5	4.9	6.0	4.9	5.6	6.3	5.1	5.6	5.1	5.3
0.6	6.8	6.1	5.2	5.9	5.5	5.2	6.3	5.8	6.4
0.7	7.4	5.8	5.9	6.2	6.5	6.2	6.5	5.7	5.8
0.8	6.7	8.6	7.4	7.8	7.1	7.0	6.1	7.4	6.1
0.9	6.8	7.8	8.2	7.9	7.8	7.8	8.1	7.3	7.1

Table 6. VECM estimates for car quota premium and stock price example

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}$	r_{12}	T^*
m=1	-0.191 (0.048)	-0.003 (0.007)	3.71	-0.02	104
m=2	-0.230 (0.062)	-0.016 (0.013)	3.56	-0.10	51
m=3	-0.342 (0.093)	-0.015 (0.021)	3.74	-0.02	34
m=4	-0.368 (0.109)	-0.046 (0.026)	2.79	0.12	25
m=5	-0.483 (0.133)	-0.027 (0.036)	3.06	-0.04	20
m=6	-0.572 (0.095)	-0.088 (0.044)	3.25	-0.01	16

T^* is the effective sample size. The numbers in parentheses are standard errors.