On being balanced in an unbalanced world

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Published in:
Journal of the Operational Research Society

DOI:
10.1057/jors.2012.29

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ABSTRACT

This paper examines the case of a procurement auction for a single project, in which the breakdown of the winning bid into its component items determines the value of payments subsequently made to bidder as the work progresses. Unbalanced bidding, or bid skewing, involves the uneven distribution of mark-up among the component items in such a way as to attempt to derive increased benefit to the unbalancer but without involving any change in the total bid. One form of unbalanced bidding for example, termed Front Loading (FL), is thought to be widespread in practice. This involves overpricing the work items that occur early in the project and underpricing the work items that occur later in the project in order to enhance the bidder's cash flow. Naturally, auctioners attempt to protect themselves from the effects of unbalancing - typically reserving the right to reject a bid that has been detected as unbalanced. As a result, models have been developed to both unbalance bids and detect unbalanced bids but virtually nothing is known of their use, success or otherwise. This is of particular concern for the detection methods as, without testing, there is no way of knowing the extent to which unbalanced bids are remaining undetected or balanced bids are being falsely detected as unbalanced.

This paper reports on a simulation study aimed at demonstrating the likely effects of unbalanced bid detection models in a deterministic environment involving FL unbalancing in a Texas DOT detection setting, in which bids are deemed to be unbalanced if an item exceeds a maximum (or fails to reach a minimum) “cut-off” value determined by the Texas method. A proportion of bids are automatically and maximally unbalanced over a long series of simulated contract projects and the profits and detection rates of both the balancers and unbalancers are compared.

The results show that, as expected, the balanced bids are often incorrectly detected as unbalanced, with the rate of (mis)detection increasing with the proportion of FL bidders in the auction. It is also shown that, while the profit for balanced bidders remains the same irrespective of the number of FL bidders involved, the FL bidder's profit increases with the greater

proportion of FL bidders present in the auction. Sensitivity tests show the results to be generally robust, with (mis)detection rates increasing further when there are fewer bidders in the auction and when more data is averaged to determine the baseline value, but being smaller or larger with increased cut-off values and increased cost and estimate variability depending on the number of FL bidders involved. The FL bidder's expected benefit from unbalancing, on the other hand, increases when there are fewer bidders in the auction. It also increases when the cut-off rate and discount rate is increased, and when there is less variability in the costs and their estimates, and when less data is used in setting the baseline values.

BACKGROUND

This paper examines the case of a procurement auction for a single project, in which the breakdown of the winning bid into its component items determines the value of payments subsequently made to the bidder as the work progresses. Unbalanced bidding, or bid skewing as it is sometimes termed in the case of timber-sale auctions (e.g.,, Rothkopf & Harstad 1994; Athey & Levin 2001; Haley & Dunphey 2010), involves the uneven distribution of mark-up among a bid's component items in such a way as to attempt to derive increased benefit to the unbalanced bidder but without involving any change in the total tender price[1]. It contributes to efficient resource utilization and ultimately benefits clients "… since it helps to ensure that the most efficient constructor constructs" (Stark 1974:377). Amongst other things, it offers an important means of reducing the contractor's debt incurred in providing resources in advance of 'in-progress' payments from the client/owner, as this can be substantial[2] (Kenley, 2003).

Its use appears to be ubiquitous. As far back as 1935, unbalanced bidding was reported to be "frequently practiced" although "much less flagrantly" than before 1920! (Chawner 1935: 576). By 1974, unbalancing was said to be "virtually universal" (Stark 1974: 377) and since then "common" (Clough & Sears 1994; Hinze 1993). Most recently, unbalanced bidding is thought to

[1] Following Stark (1974: 373) these terms are used interchangeably: bidding, tendering; unit price, unit bid, bill rate; bidder, contractor; auctioner, seller, owner, client, sponsor; Unit Price Proposal, Bill of Quantities; unbalancing, loading of bill rates; front loading, front end loading.

[2] As Stark notes, some construction projects, such as highways and maintenance are "more a challenge of finance than of technology" (Stark 1974:375)

be "the most widely used tactic" for making the bid price more competitive ahead of unexpected markdown, loss-first-profit-later quotation, multiple alternative quotations, etc. (Liu *et al* 2009: 188).

One of the various alternative forms of unbalancing is "back-end loading", which involves applying relatively high prices for items scheduled for *late* completion, and particularly those items that are expected to have a high rate of inflation, and proportionately lower prices for other items, in order to gain extra payments where compensation of inflation is being awarded (Cattell, 1987). Another is "Front loading" (FL), which involves apportioning relatively high prices to items scheduled for *early* completion and proportionately lower prices for other items in order to improve cash flow in the early stages of the project. FL is considered to be the most common type of unbalancing (Arditi & Chotibhong 2009: 721; Christodoulou 2008: 1293), with Stark (1974) claiming it is used in "virtually every tender".

The practice of FL comes as a likely added cost to clients. If it succeeds in producing an improved cash flow for the bidder, this comes at the expense of a worse cash flow for the client. Furthermore, besides having to pay the bidder sooner than would otherwise be necessary, the client is also having to take the risk that the bidder will not default mid-way through the project. If that were to happen, the client will be left with a situation where they will have overpaid for the interim work that will have been done and therefore unlikely to find another bidder willing to complete the project for the amount left over.

However, there is little empirical evidence for these claims. Stark's experience is limited to just one major UK construction company. Only Green (1986) has attempted to survey the field – with a series of interviews involving three contractors and three consultants – finding that two of the contractors used FL and "considered the practice [of unbalancing] to be widespread, contrary to the consultant's views" (Tong & Lu 1992: 70). It should be said of course, that the lack of evidence is unsurprising, with FL (and unbalancing in general) thought by many to be an unethical practice (e.g., Arditi & Chotibhong 2009).

According to Stark (1974), FL in practice is often “neither competent nor conscious”, and several models have been presented as a means of systemising the process. Gates (1959, 1967) seems to have been the first to set out the steps involved, followed by Stark’s (1968, 1972, 1974) reformulation as a linear program - claimed to be automatic as “the solutions to the appropriate linear programs simply are not within the unaided human capacity” (Stark 1974: 387). Later Doersch & Patterson (1977), Ashley & Teicholz (1977) and Tong and Lu (1992) propose different versions of Stark’s original deterministic model, with Diekmann *et al* (1982), Cattell (1984, 1987), Christodoulou (2008), Afshar and Amiri (2008) and Liu (2009) developing non-deterministic alternatives. Also, Cattell *et al* (2008) have shown that FL models can be significantly simplified by having the objective of maximizing a project’s top-line revenue rather than maximizing bottom-line profit. A test done on a hypothetical project by Cattell (2009) indicated significant benefits, in terms of increased profits without necessarily increased risks. However, no empirical tests of these models have been reported as yet and it is recommended that contractors should use sensitivity analysis to assess their robustness (Stark 1974: 5).

Of course, FL is not without its risks. Clients typically perceive unbalanced bids as being more costly to them. The main concern for the bidder is that the bid may be deemed ineligible. A situation reported in Taiwan is that the bidder is asked to explain unbalanced-looking bids with a view to a negotiated adjustment (Wang 2004). Typical UK practice is similar, with the client often reserving the right not to accept the lowest or any bid. In other cases, a “blatantly unbalanced” bid may be refused (Doersch & Patterson 1977: 883). USA practice seems to be dominated by the various federal and state ‘regulations’, where “irregular” bids such as those that are FL can be disqualified or rejected (Stark, 1974; Christodoulou 2008; Arditi & Chotibhong 2009). The California Department of Transport (CalTran), for example, have recently been reported as awarding four percent of their contracts to other than the lowest bidder and, according to industry sources, most likely due to unbalanced bids (Bajari *et al* 2006: 9). CalTran may also insist on renegotiating unit prices and are considering the possibility of a more flexible penalty function (Bajari *et al* 2006: 9).

Correctly applying these penalties, however, relies on the accurate detection of unbalanced bids by the client but, as has been noted, “... very few researchers have explored the ways of preventing unbalanced bids" (Arditi & Chotibhong 2009: 271). Current UK practice is for a consultant (quantity surveyor) to "look at the item prices to see if they are free from arithmetic error, reasonable (with respect to conformity with industry standards) and not likely to distort the contract in a manner that is to the client's disadvantage" (Cattell 1984: 7). The practice in the British Commonwealth countries is similar. One major quantity surveying practice in Hong Kong, for example, regularly assesses unit prices for the lowest overall bid by comparing them with the unit prices for the lowest three competing bids (Cheung 2010).

A survey of USA practice in detecting unbalanced highway construction bids was conducted in 2004 by the American Association of Highway and Transportation Officials (AASHTO) and summarised in Arditi & Chotibhong (2009). Many of the state departments of transport (DOTs) involved attempt to identify unbalanced bids without any formal procedures. Those with formal procedures use either an "engineer's estimate" or average of unit prices as a baseline figure from which to judge individual unit prices. "Irregular bids" are then determined by their distance from this baseline by a variety of percentage "cut-offs" (in which an item exceeds a maximum, or fails to reach a minimum, value determined by the detection method) and formulae used.

Wang (2004) and Arditi & Chotibhong (2009) have attempted to "automate the process" involved in detecting unbalanced bids. Wang's procedure is aligned to the Taiwan system, termed the *owner-based approach*, where the baseline unit rates are provided by the owner – adjusted by the lowest bid/owner total ratio (discounting ratio) "employed under the belief that the owner's cost estimate is more reasonable than the bidder because his cost estimate is more thoroughly prepared by the architect" (Wang 2004: 455). This is then embodied in a spreadsheet to identify unit prices exceeding ±30% of the owner's rates. Arditi & Chotibhong (2009) procedure is also based on a spreadsheet and follows the USA approach by identifying unit prices exceeding a similar percentage difference to the engineer's estimate/average bid baseline value. Although they claim the model to be "fully automated", it does still require some input in order to operate in addition to the bidders' data. That is, an approximate payments schedule, the engineer's baseline estimates of the unit prices (if applicable), the percentage cut-off values and discount rate.

Cattell et al (2007), in reviewing the various models suggested for use by bidders, have been particularly critical of the arbitrary nature of the cut-off values as, “despite their significance, it is commonly recommended that they should be decided upon without any scientific or mathematical aid”. Of course, the real problem here is that there is no data available to enable such a scientific aid to be developed for, as noted earlier, the risks involved in unbalancing are such that it is not in the interests of anyone involved to admit to the practice. In fact, the lack of any real data on the nature and extent of unbalancing in practice in the industry raises fundamental questions concerning the efficacy or otherwise of any method aimed at detecting unbalanced bids that relies on existing price data. If, as is believed by many, the practice of unbalancing is indeed “virtually universal”, the baseline values themselves will be more representative of unbalanced bids than balanced bids. Where the baseline values comprise the simple average unit price of the bids, and the majority of bids are unbalanced, the baseline figure itself will be equally unbalanced. Likewise, even in the situation where the engineer’s estimate provides the baseline figure, an engineer continually and unknowingly exposed to unbalanced bids is unlikely to realise that they are indeed unbalanced and will instead treat them as balanced. In such a situation, therefore, it is likely that the unbalanced bids will appear to be balanced and the balanced bids appear to be unbalanced – outcomes termed type I and type II errors in statistical hypothesis testing, where a type I error is defined as the situation in which a correct hypothesis is inappropriately rejected while a type II error occurs when a false hypothesis is inappropriately retained.

In this paper, we simulate the effects of these equivalent type I and II errors for situations where there are a few balanced bidders in a world of many unbalanced bidders. Arditi & Chotibhong’s (2009) (A-C) detection system is used, with baseline values derived by two means: (1) their original method of averaging the unit prices provided by the bids for a single contract and (2) the use of a surrogate engineer’s estimate derived by averaging all unit prices for a database of previously bid contracts. For the unbalancing process itself, we assume only FL is used and follow the deterministic method in also assuming each FL bidder knows the cut-off values for each unit price. The resulting profit levels and rejection rates are provided for *both* FL and balanced bidders and which show that, under the conditions assumed, the type I and type II errors increase significantly as the proportion of FL to balanced bidders increases – with the FL bidders gaining substantial profit at the expense of the balanced bidders and the balanced bidders being increasingly (mis)identified as unbalanced. Sensitivity analysis confirms the results to be robust for a range of bidders, averaging methods, cut-off values, discount rates and cost and estimating variability.

Notice that construction work is of such a nature that the costs that contractors incur, building the same item of work, are not identical, even though the specification of the item is prescribed. Furthermore, contractors have to estimate these costs when pricing their work, and there's an inherent inaccuracy in these estimates. Beeston (1975) noted that the variance between contractors' prices for component items is substantially more than the variance between their composite bids for overall projects. These underlying variances in costs and in estimating accuracy are shown, in this simulation, to be substantial contributors to the misidentification of both balanced and unbalanced bids.

Model

Based on Cattell et al (2008), the profit present value is given by

$$PV = \sum_{i=1}^{n} \left(\frac{1}{1+r_i} \right)^{y_i} q_i \left(p_i - c_i \right) \quad \mathbf{(1)}$$

where $i =$ item number

$n =$ number of items

$r_i =$ discount rate for item i

$y_i =$ year of payment due for item i (payment schedule)

$q_i =$ number of units (quantity) of item i

$c_i =$ actual unit cost of item i, where $c_j \sim N(\mu_{c_j}, \sigma_{c_j})$

$p_i =$ bill price per unit of item i, where $p_i = c_i' \left(1 + \frac{m_i}{100} \right)$ for the balanced bidder

$c_i' =$ unbiased estimated cost of item i, where $c_i' \sim N(\mu_{c_i}, \sigma_{c_j}')$

$m_i =$ percent markup for item i, where $m_i \sim N(\mu_{m_i}, \sigma_{m_i})$

For unbalanced bidding, p_i is a strategic value which may or may not be a function of c_i' and m_i depending on how the detection cut-off value is derived. In the deterministic situation, the optimal value of p_i is the cut-off value provided by the detection method in use – where $p_i =$ the maximum cut-off value for those items that occur early in the project and $p_i =$ the minimum cut-off value for those items that occur later in the project. [Martin, is this a problem with the conversion from docx at my end, that there are multiple and conflicting references to p_i – for both min and max?]

SIMULATION STUDY

Standard model

For the purposes of the simulation study, a standard model was adopted. This is for a single contract comprising Gates' (1967) four items as shown in Table 1[3]. To these were added a notional mean actual cost (μ_{c_i}), mean mark-up value (μ_{m_i}), payment date (y_i), and the standard deviations of actual cost (σ_{c_i}), estimated cost (σ_{c_i}') and mark-up (σ_{m_i}). The payment schedule indicates the year in which payment for the item is due so, for example, a value of 0 indicates payment is due immediately, while a value of 0.5 indicates payment is due midyear, etc.

Also for the standard model, 10 bidders are assumed and a 15% discount rate $(r_i = 0.15)$. The Texas DOT cut-off criteria apply. That is, a range of 100% above or 50% below the baseline for major items, and 200% above or 75% below the baseline values for minor items, with major items being defined as items that cost more than 5% of the contract value (Arditi & Chotibhong 2009: 724). For the standard model, the baseline value of an item for a contract was represented by the average unit price for that item for that contract.

[3] This is to be compared to real-world conditions, where a project might comprise thousands of items.

A simulation run consisted of 10,000 simulations of this single contract. Therefore, for the contract 1, the value of bidder 1's actual cost for item 1 was simulated by a random number generated from N($\mu = 50000$, $\sigma = 5000$) and estimated cost also from N($\mu = 50000$, $\sigma = 5000$), with a mark-up from N($\mu = 10$, $\sigma = 5$). This was repeated for all 10 bidders for contract 1. This whole process was repeated again for the remaining 9,999 contracts.

An FL unbalanced bid was therefore simulated as follows:

1. A set of 10 balanced bids for contract 1 was simulated for the standard model as above
2. The average price was calculated for the 10 balanced bids for item 1 for this contract
3. The engineer's estimate was set as the value of 2 above
4. The Texas cut-off was then calculated for item 1 based on the value of 3 above
5. 2-4 was repeated for each of the remaining items
6. To maximise profit by FL unbalancing, bidder 1's unit prices were reset to the maximum cut-off for the early schedule items and minimum cut-off for the later schedule items.
7. The average price was recalculated for the 9 balanced bids and 1 unbalanced bid for item 1 for this contract and steps 3-7 above were repeated for each bidder until convergence.

For multiple FL bidders, steps 2-7 above were repeated for each bidder until convergence.

PV profit and rejection rates

The present value profit was calculated for each bidder and each contract for the standard model. This is the difference between the item price and item actual cost discounted by the 15% discount rate. Fig 1a shows the results for the standard model for the FL bidders in the presence of 0(1)10 FL bidders (the result where 'number of FL bidders'=0 is the PV profit for the balanced bidders). This indicates a steady increase in PV profit for the FL bidders as their numbers increase. The PV profit for the balanced bidders remains constant irrespective of the number of FL bidders. However, this is not the case for the rejection rates. Of course, being a deterministic model, the rejection rate for the FL bidders is zero, as they are assumed to know the cut-off values and therefore have no risk of exceeding them. The balanced bidders, on the other hand, increasingly exceed the cut-off values as shown in Fig 1b. This indicates the percentage of bids made by a balanced bidder in which at least one item exceeds the cut-off value for that item. As is shown, upon reaching the point where there are eight FL out of the ten bidders, *all* of the balanced bidders are incorrectly identified as unbalanced.

Sensitivity

Several sensitivity tests were carried out to assess the robustness of these results. Firstly, the standard model was modified from 10 bidders to a number of bidders ranging from two to 10. Figs 2a and 2b give the results, showing that the effect is *greater* where smaller number of bidders is involved both in terms of FL bidder profit and balanced bidder (mis)detection rate. Next the cut-off was changed systematically from the Texas method used in the standard model to a simple ±20(10)100% to include Wang (±30%) and Arditi & Chotibhong's (±20-25%) values. This also has an increased effect of the FL bidders' profit, depending on the number of FL bidders involved (Fig 3a). The mis(detection) rate for the balanced bidders, however, is less uniform with the rate reducing with larger cut-off percentages where less FL bidders are involved and increasing with a change in profile where more FL bidders are involved (Fig 3b).

Increasing the discount rate, as expected, has the effect of increasing FL bidder profit but with no effect at all on the (mis)detection rate (Figs 4a and 4b). Figs 5a and 5b show the effects of changing the variability of the costs and estimated costs, with a reduced FL profit where there is greater variability but a relatively greater (mis)detection rate where there are fewer FL bidders involved and less where there are more FL bidders involved.

Finally, a surrogate engineer's baseline value was used in comparison with the average bid method. To do this, a surrogate value was used of the average unit price of items in the database of 10,000 contracts as follows:

1. A set of balanced bids for all 10,000 contracts was simulated as previously described
2. The average unit price was calculated for item 1 for the n lowest overall bidders for each contract for all contracts
3. The engineer's estimate was set as the value of 2 above

4. The Texas cut-off was then calculated for item 1 based on the value of 3 above
5. 2-4 was repeated for each item
6. To maximise profit, bidder 1's unit prices were reset to the maximum cut-off for the early schedule items and minimum cut-off for the later schedule items.
7. 2-6 above was repeated until convergence as before.
8. Again, for multiple FL bidders, steps 2-7 above were repeated for each bidder until convergence.

Figs 6a and 6b show the results of using the surrogate engineer's baseline value when using the data for the lowest bidder (EE1) and the averaged data for the lowest three bidders (EE3), showing that the FL bidder profit reduces slightly as more bidder's data is used to average the surrogate engineer's estimate, while the (mis)detection rate increases as more data is used.

CONCLUSIONS

The practice of bid unbalancing seems to have originated a long time ago, being reported as "flagrant" even before the 1920s. Since Gates' (1967) enunciation of the steps involved in unbalancing, followed by Stark's (1972) LP formulation, several researchers have proposed means of better systemising the process to the point of automation. Likewise, work on systemising its detection also seems to be reaching a similar point. Meanwhile, empirical work on establishing the frequency of *actual* unbalancing has been limited to very few small samples – making the methods of detection of doubtful value as there appears to be no way of knowing whether the methods actually detect, or fail to detect, unbalanced bids when they are really unbalanced, or falsely detect unbalanced bids when they are really balanced.

The study reported in this paper aimed to demonstrate the results of using a typical method of detection under some simple assumptions, such as that most unit prices are unbalanced and therefore methods that use existing unit prices to test for unbalancing are biased. To do this, both FL unbalancing methods and their detection methods have been extended to become fully automatic and then tested under deterministic, simulated conditions.

The results show that, as expected, the balanced bids are often incorrectly detected by the Texas DOT method as unbalanced, with the rate of (mis)detection increasing with the proportion of FL bidders in the auction. It is also shown that, while the profit for balanced bidders remain the same irrespective of the number of FL bidders involved, the FL bidder's profit increases with the greater proportion of FL bidders present in the auction. Sensitivity tests show that the (mis)detection rate increases further where there are fewer bidders in the auction and where more data is averaged to determine the baseline value, but also being smaller or larger with increased cut-off values and increased cost and estimate variability depending on the number of FL bidders involved. The FL bidder's profit, on the other hand, increases where there are fewer bidders in the auction, where the cut-off rate and discount rate is increased, where there is less variability in the costs and their estimates, and less data is used in setting the baseline values. Also, the nature of the situation in construction procurement bidding is that much of the variability involved is generally considered to be due to roughly symmetrical "errors" in prediction and therefore lends itself to modelling by the normal, lognormal or beta-looking distributions. With the variances involved, it is not expected that the results are sensitive to this.

This simulation is focussed on the misinterpretation of balanced bids as unbalanced bids in a deterministic setting. In real-world conditions, where bidders have no knowledge of the method or of the extent to which any detector will impose limits when testing their bid, it is hypothesised that, equally, unbalanced bids will be misinterpreted as balanced bids. This is yet to be tested however and needs to be addressed in future work. Similarly, the four items from Gates' early work on unbalancing, while providing a simple and easy to understand example of the basic issues involved, need to be extended to a treatment of a more real-world situation containing many more such items. Further work is therefore also needed to develop this aspect in a more comprehensive and realistic setting.

Meanwhile, the results to date caution against the use of untested methods for detecting strategic bidding such as unbalancing. If, as Stark asserts, unbalanced bidding is "virtually universal" then a balanced bid, if such exists, is likely to look strange. In the words of Rousseau (1756: 91), "to be sane in a world of madmen is itself a kind of madness".

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Item	$i=1$	$i=2$	$i=3$	$i=4$
Description	Clearing	Earth excavation	Rock excavation	Cleaning up
Unit	Lump sum	Cubic yards	Cubic yards	Lump sum
Quantity (q_i)	1	50,000	25,000	1
Mean actual unit cost (μ_{c_i})	50,000	1.50	3.00	50,000
Actual cost sd (σ_{c_i})	5,000	0.15	0.3	5,000
Estimated cost sd (σ'_{c_i})	5,000	0.15	0.3	5,000
Mark-up mean (μ_{m_i})	10%	10%	10%	10%
Mark-up sd (σ_{m_i})	5	5	5	5
Payment due (y_i)	0	0.5	0.5	1

Table 1: Gates' four item contract

Fig 1a: Profit pv% for a FL bidder (standard model)

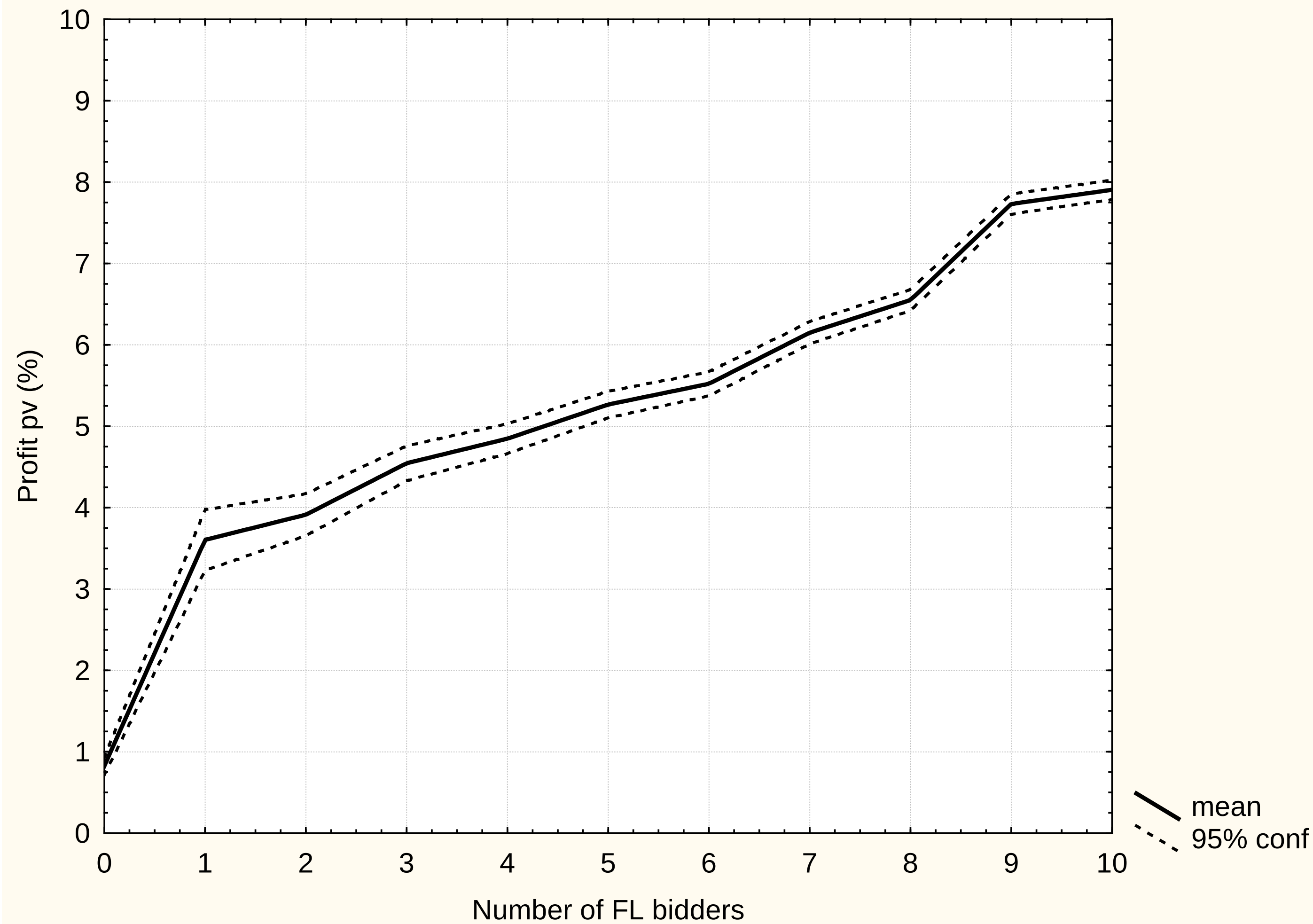
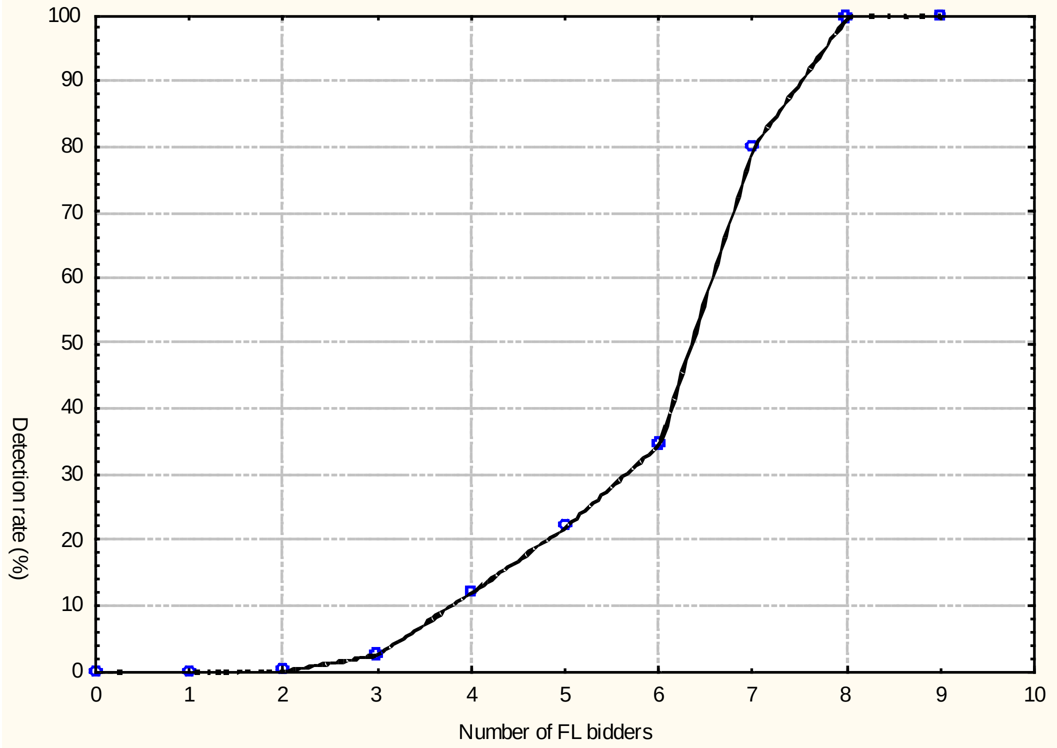


Fig 1b: Avg detection rate for balanced bidder (standard model)



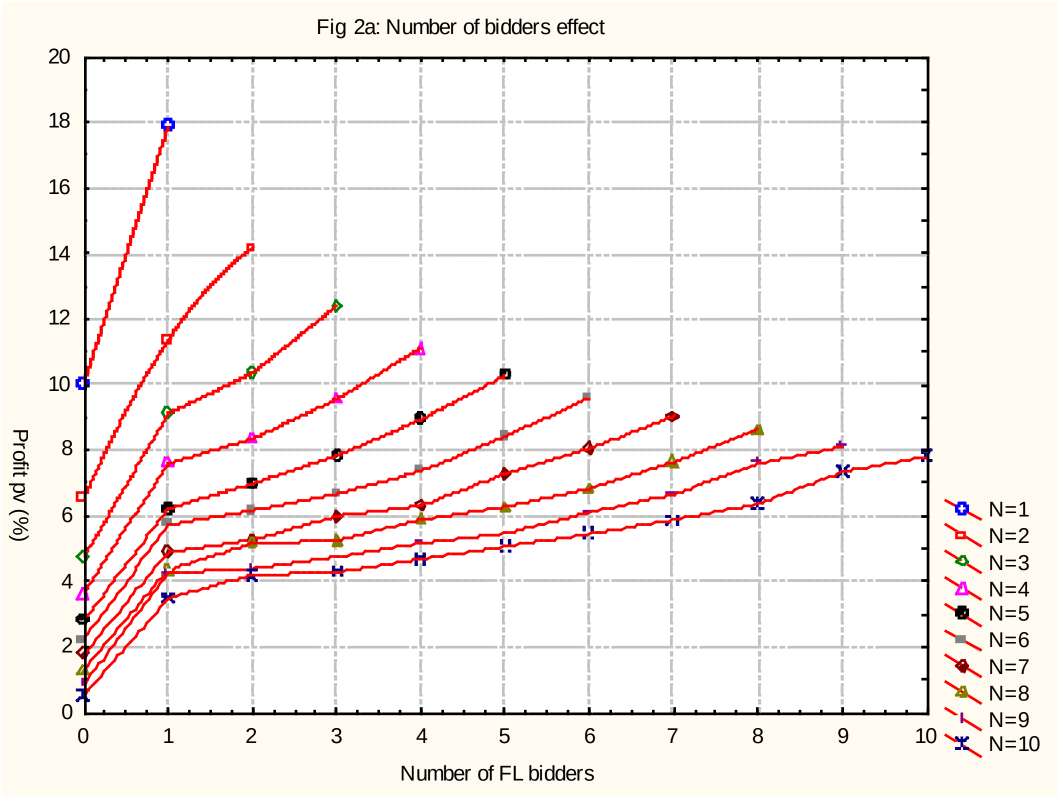


Fig 2a: Number of bidders effect
Profit pv (%)
Number of FL bidders
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8
9
10
N=1
N=2
N=3
N=4
N=5
N=6
N=7
N=8
N=9
N=10

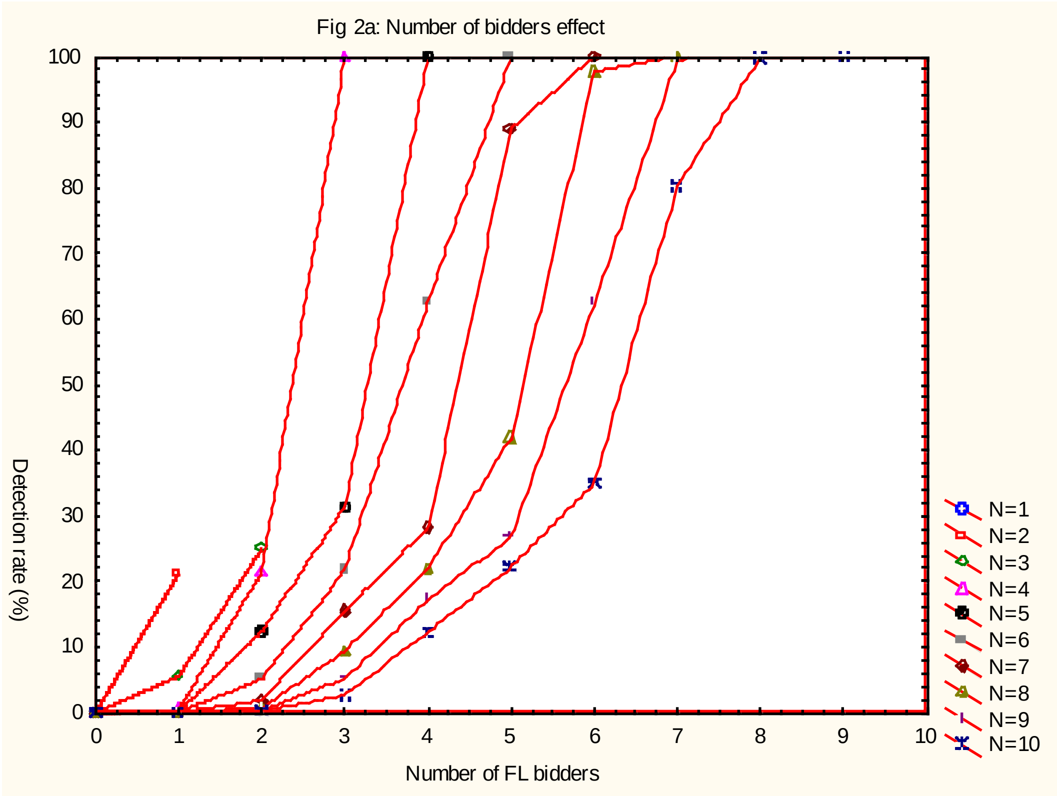


Fig 2a: Number of bidders effect
Detection rate (%)
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
6
7
8
9
10
Number of FL bidders
N=1
N=2
N=3
N=4
N=5
N=6
N=7
N=8
N=9
N=10

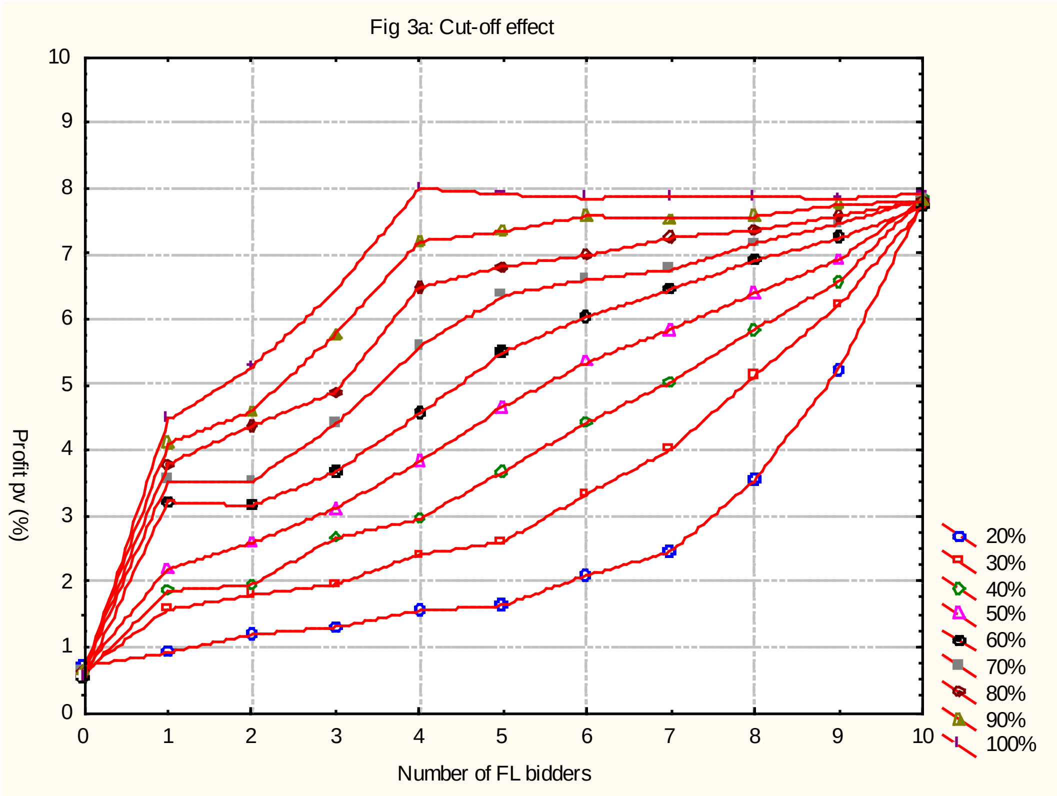


Fig 3a: Cut-off effect
Profit pv (%)
Number of FL bidders
0
1
2
3
4
5
6
7
8
9
10
20%
30%
40%
50%
60%
70%
80%
90%
100%

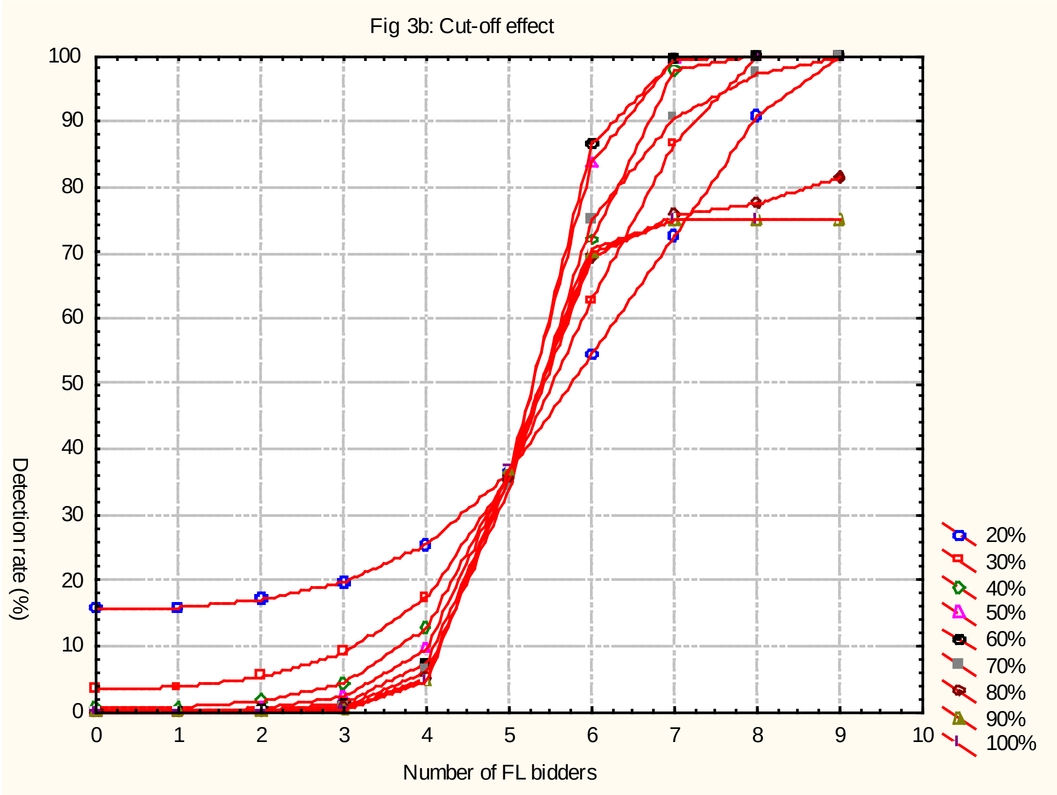


Fig 3b: Cut-off effect
Detection rate (%)
Number of FL bidders
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
6
7
8
9
10
20%
30%
40%
50%
60%
70%
80%
90%
100%

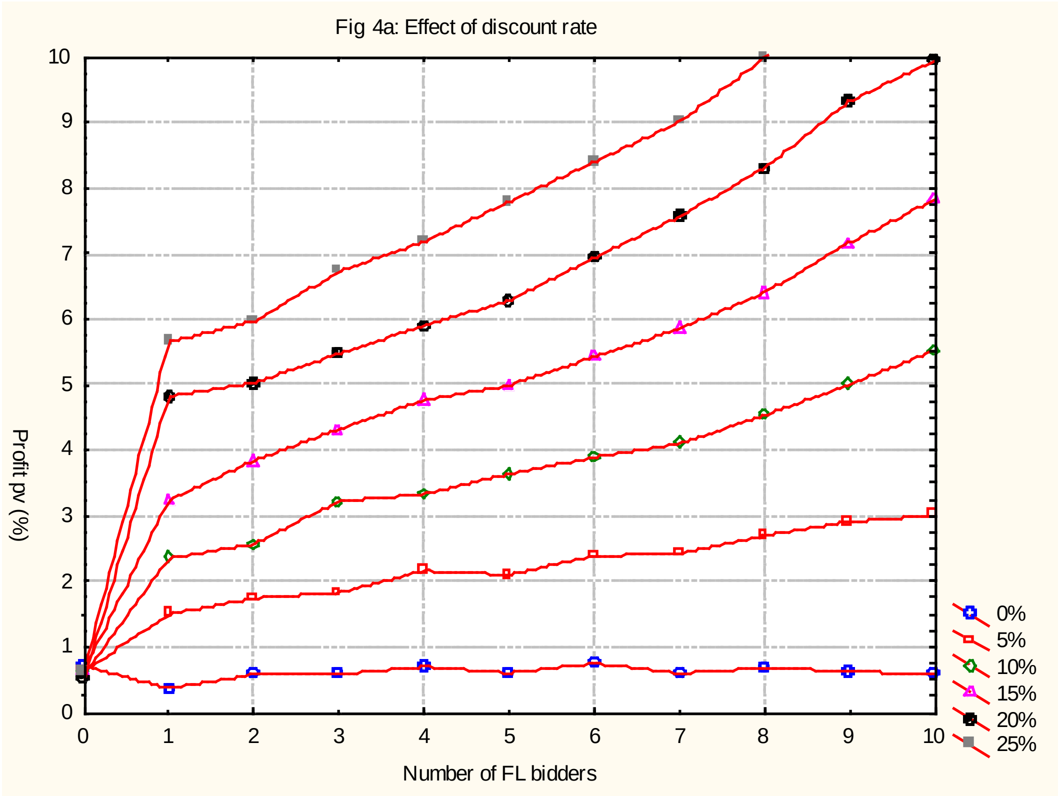


Fig 4a: Effect of discount rate
Profit pv (%)
Number of FL bidders
0%
5%
10%
15%
20%
25%
0 1 2 3 4 5 6 7 8 9 10

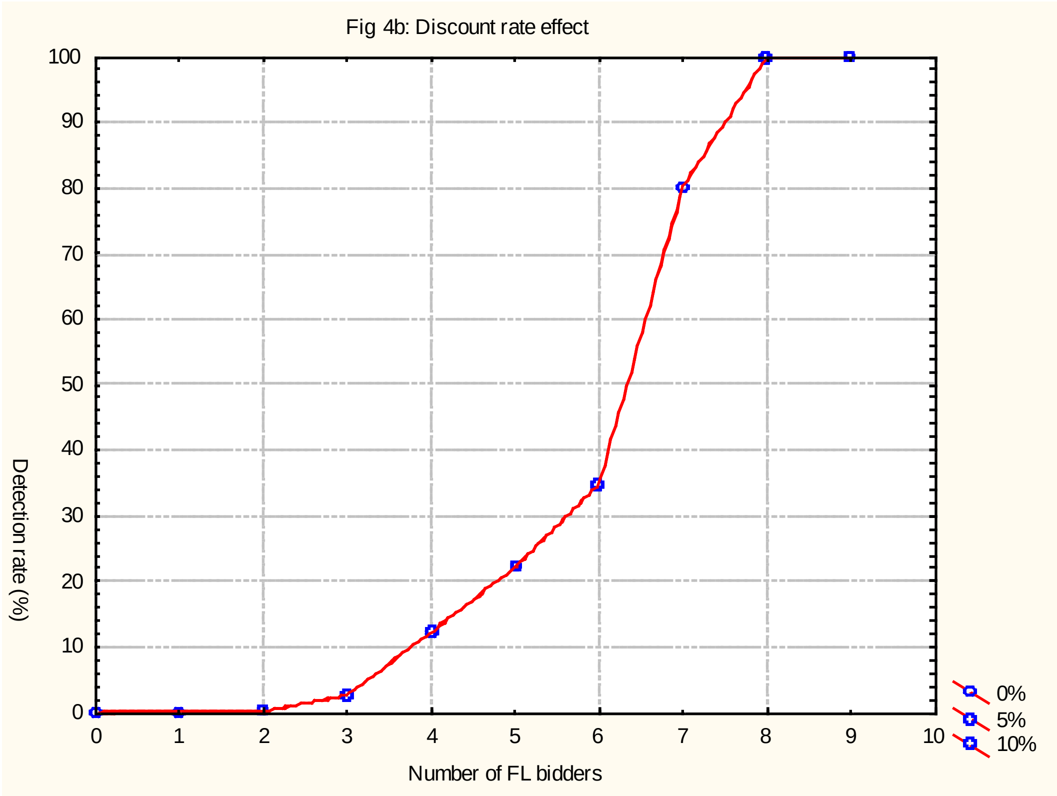


Fig 4b: Discount rate effect
Detection rate (%)
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
Number of FL bidders
0%
5%
10%

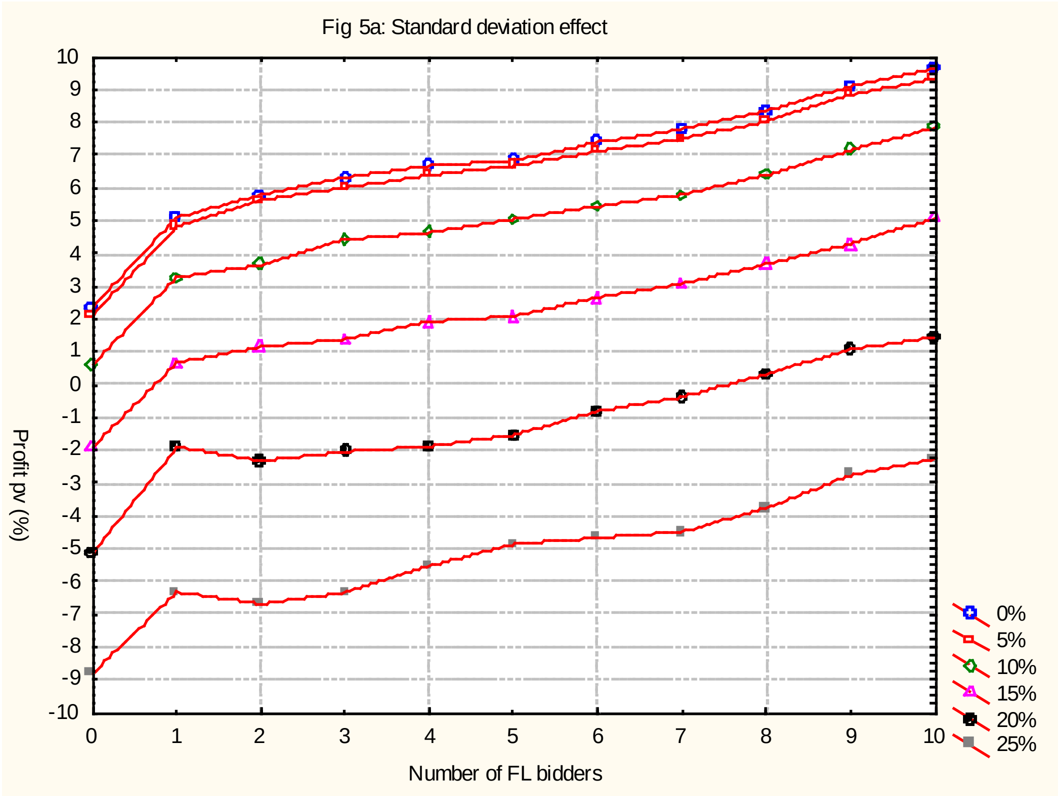


Fig 5a: Standard deviation effect
Profit pv (%)
Number of FL bidders
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
0
1
2
3
4
5
6
7
8
9
10
0%
5%
10%
15%
20%
25%

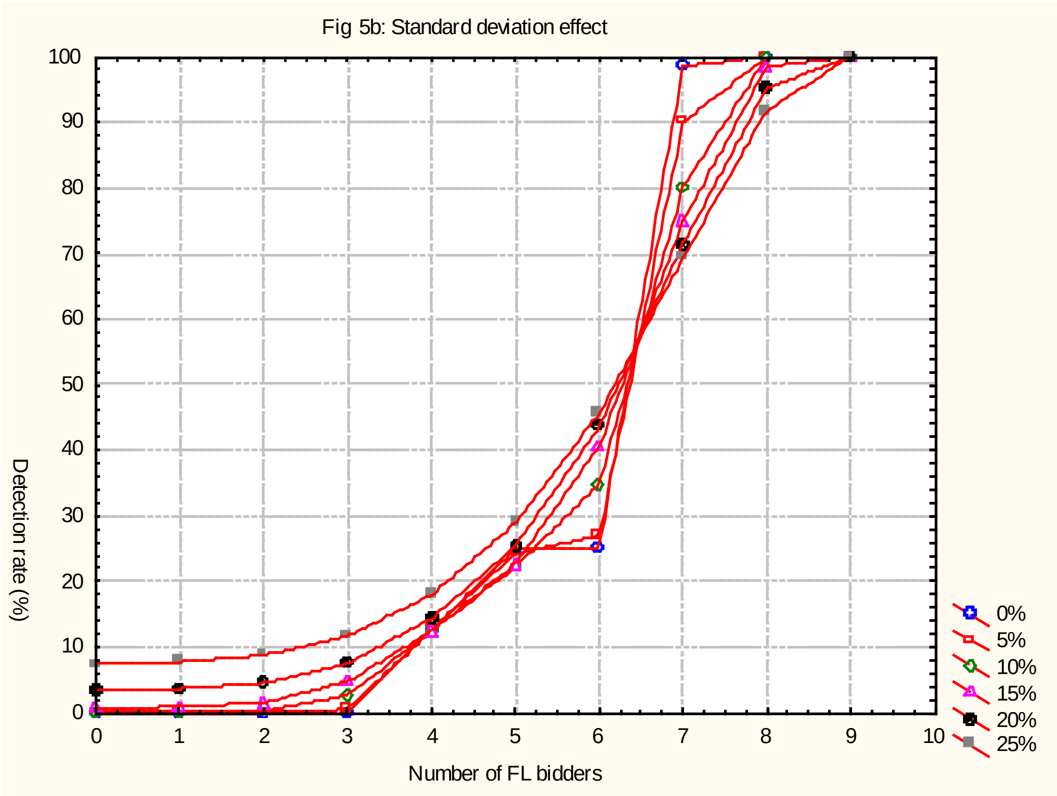


Fig 5b: Standard deviation effect
Detection rate (%)
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
Number of FL bidders
0%
5%
10%
15%
20%
25%

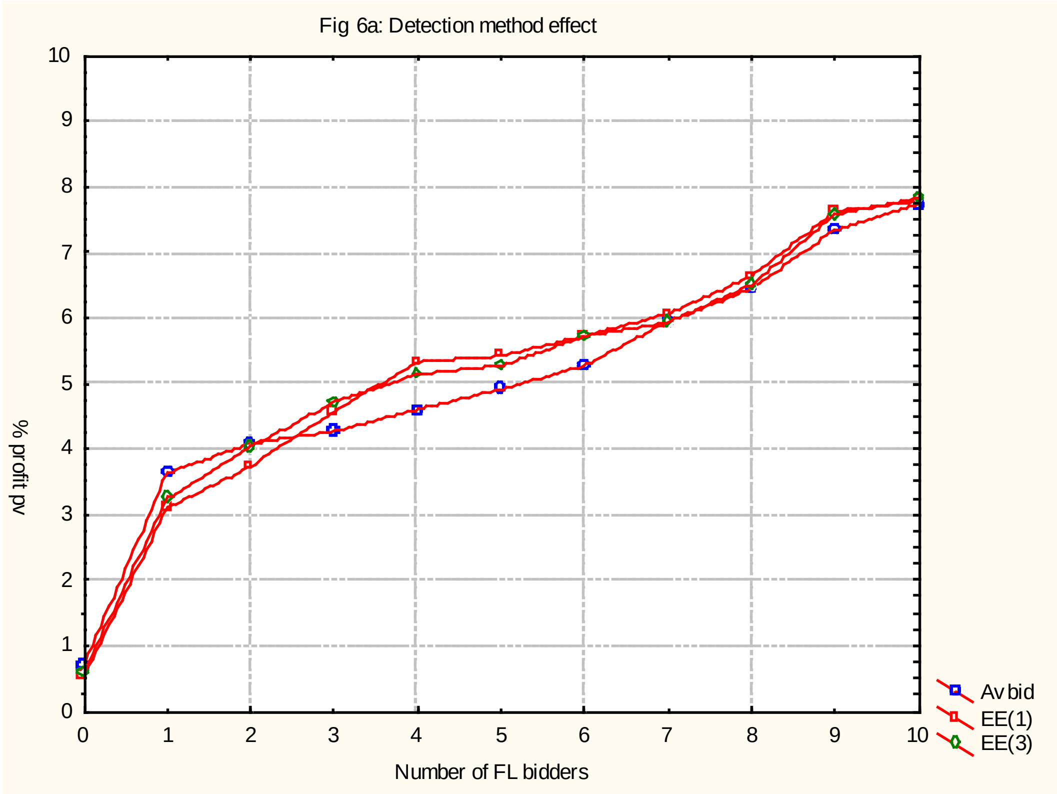


Fig 6a: Detection method effect
% profit pv
Number of FL bidders
0
1
2
3
4
5
6
7
8
9
10
Av bid
EE(1)
EE(3)

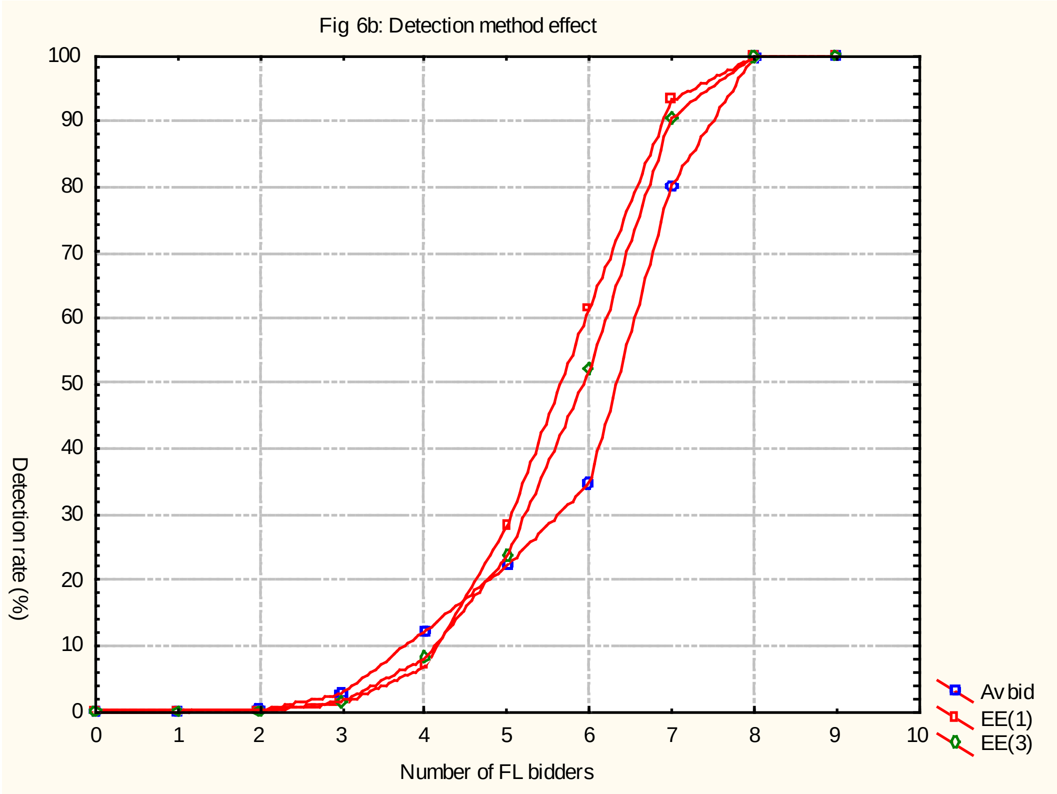


Fig 6b: Detection method effect
Detection rate (%)
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
Number of FL bidders
Av bid
EE(1)
EE(3)