The Effects of Temporal Aggregation on Granger Causality

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2001 Econometric Society Australasian Meetings

THE EFFECTS OF TEMPORAL AGGREGATION ON GRANGER CAUSALITY

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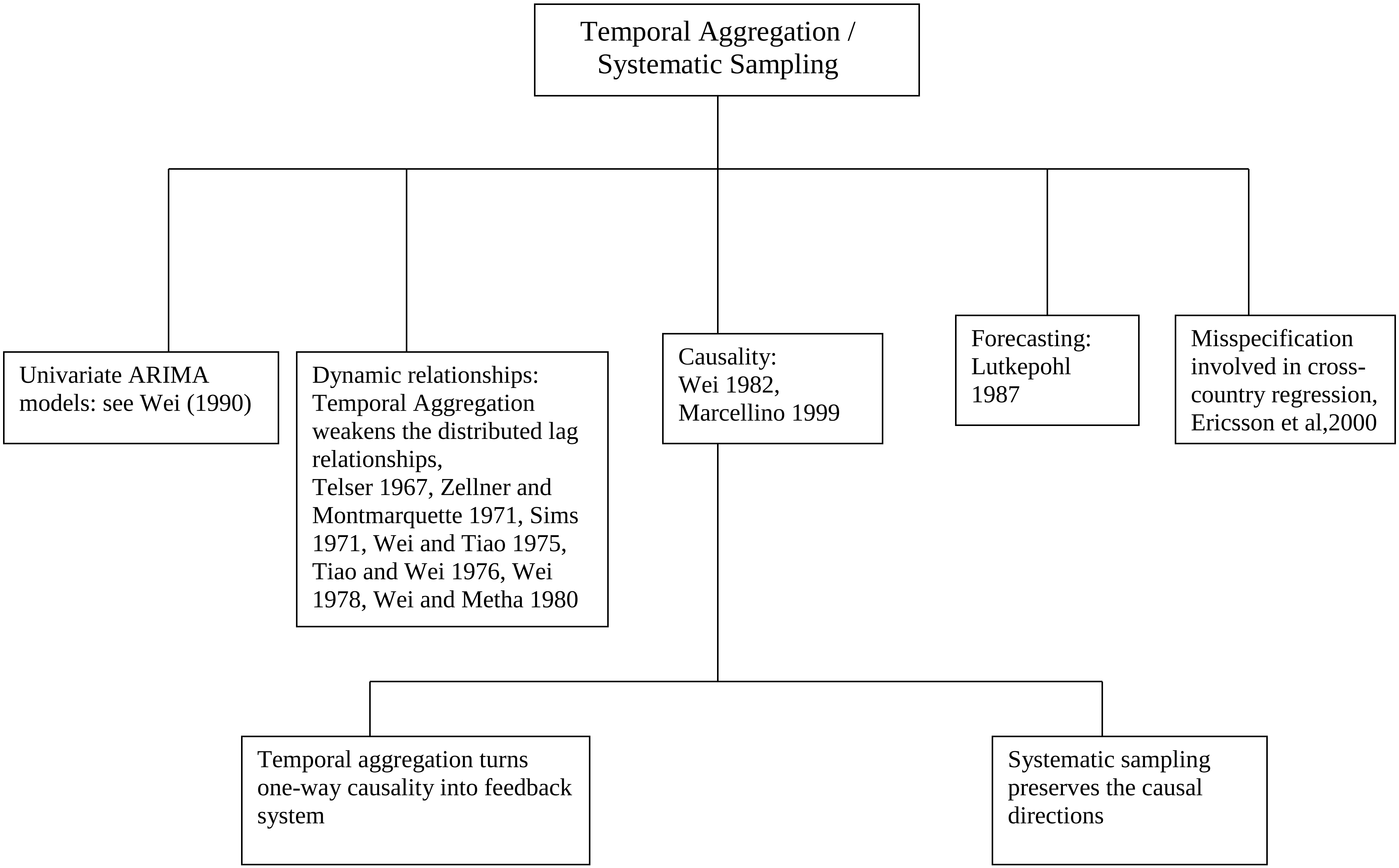
March 2001

Introduction

- Use of highly temporally aggregated data for causal inference is quite common in applied econometric literature
    - For example, Jung and Marshall 1985, Rao 1989, Demitriades and Hussein 1996 use highly aggregated data to find Granger causality between economic growth and export growth, economic growth and trade, and economic growth and financial development
    - On the other side are those who use cross-country regressions with data averaged over many years to establish the contemporaneous relationship among the variables of interest (for example, Barro 1991, Levine and Renelt 1992, King and Levine 1993, Levine and Zervos 1993, Frankel and Roamer 1999)
- The objective of this paper is to examine how temporal aggregation affects causal relationships among variables


Temporal Aggregation / Systematic Sampling
Univariate ARIMA models: see Wei (1990)
Dynamic relationships: Temporal Aggregation weakens the distributed lag relationships, Telser 1967, Zellner and Montmarquette 1971, Sims 1971, Wei and Tiao 1975, Tiao and Wei 1976, Wei 1978, Wei and Metha 1980
Causality: Wei 1982, Marcellino 1999
Forecasting: Lutkepohl 1987
Misspecification involved in cross-country regression, Ericsson et al,2000
Temporal aggregation turns one-way causality into feedback system
Systematic sampling preserves the causal directions

RELATIONSHIP BETWEEN CROSS COVARIANCES OF DISAGGREGATE AND AGGREGATE SERIES

- Let $z_t = (z_{1t}, z_{2t}, \ldots, z_{nt})$ be a vector of basic disaggregate series and Z_t be the temporally aggregated vector.
- Temporal aggregation involves the construction of non-overlapping sums that can easily be obtained by defining the overlapping sum $X_t = (1 + L + \ldots + L^{m-1}) z_t$ and then defining $Z_t = X_{mt}$.
- Let $w_t = (1-L)^d z_t$ be a weakly stationary process with mean zero and variance covariance matrix

$$\Gamma^w(k) = E(w_t w_{t-k}) = [\gamma_{\mathrm{ij}}(k)], \quad i, j = 1,2,\ldots,n \qquad (1)$$

- Let $\mathrm{W}_\tau = (1 - L')^d Z_\tau = (1 - L^m)^d X_{m\tau} = (1 + L + \ldots\ldots + L^{m-1})^{d+1} w_{m\tau}$, where L' is the backward shift operator on the aggregate time unit τ.
- The cross covariance between $W_{i\tau}$ and $W_{j\tau-k}$ is given by

$$\gamma_{ij}^W(k) = Cov(W_{i\tau}, W_{j\tau-k}) = (1 + L + L^2 + \ldots\ldots + L^{m-1})^{2(d+1)} \gamma_{ij}^w(mk + (d+1)(m-1)) \qquad (2)$$

where L operates on the index of $\gamma^w_{ij}(k)$ such that $L\gamma^w_{ij}(k) = \gamma^w_{ij}(k-1)$.

⇒ The cross covariance between the aggregated series is simply the weighted sum of the cross covariances of the disaggregated series.

CAUSAL INFERENCE FROM TEMPORALLY AGGREGATED DATA

Consider the following stationary bivariate VAR(1) system:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}, \ \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right). \tag{4}$$

The coefficients φ_{12} and φ_{21} measure the feedback between y_t and x_t, with $\varphi_{12} \neq 0$ implying Granger causality from *x* to *y* and $\varphi_{21} \neq 0$ implying Granger causality from *y* to *x*.

The variances, autocovariances and cross-covariances of system (4) are given by

$$\gamma_{11}^w(0) = \sigma_y^2 = E(y_t y_t) = \varphi_{11}^2 \sigma_y^2 + \varphi_{12}^2 \sigma_x^2 + 2\varphi_{11}\varphi_{12}\gamma_{12}^w(0) + \sigma_1^2 \tag{5}$$

$$\gamma_{22}^w(0) = \sigma_x^2 = E(x_t x_t) = \varphi_{21}^2 \sigma_y^2 + \varphi_{22}^2 \sigma_x^2 + 2\varphi_{21}\varphi_{22}\gamma_{12}^w(0) + \sigma_2^2 \tag{6}$$

$$\gamma_{12}^w(0) = \gamma_{21}^w(0) = E(y_t x_t) = \varphi_{11}\varphi_{21}\sigma_y^2 + \varphi_{12}\varphi_{22}\sigma_x^2 + (\varphi_{11}\varphi_{22} + \varphi_{12}\varphi_{22})\gamma_{12}^w(0) \tag{7}$$

$$\gamma_{11}^w(k) = E(y_t y_{t-k}) = \varphi_{11}\gamma_{11}^w(k-1) + \varphi_{12}\gamma_{21}^w(k-1) \tag{8}$$

$$\gamma_{22}^w(k) = E(x_t x_{t-k}) = \varphi_{21}\gamma_{12}^w(k-1) + \varphi_{22}\gamma_{22}^w(k-1) \tag{9}$$

$$\gamma_{12}^w(k) = E(y_t x_{t-k}) = \varphi_{11}\gamma_{12}^w(k-1) + \varphi_{12}\gamma_{22}^w(k-1) \tag{10}$$

$$\gamma_{21}^w(k) = E(x_t y_{t-k}) = \varphi_{21}\gamma_{11}^w(k-1) + \varphi_{22}\gamma_{21}^w(k-1) \tag{11}$$

Solving (5)-(7), we get

$$\sigma_y^2 = \frac{c_3\left[\sigma_1^2(b_2c_3 - b_3c_2) - \sigma_2^2(b_1c_3 - b_3c_1)\right]}{[a_1c_3 - a_3c_1][b_2c_3 - b_3c_2] - [a_2c_3 - a_3c_2][b_1c_3 - b_3c_1]} \tag{12}$$

$$\sigma_x^2 = \frac{c_3\left[\sigma_1^2(a_2c_3 - a_3c_2) - \sigma_2^2(a_1c_3 - a_3c_1)\right]}{[b_1c_3 - b_3c_1][a_2c_3 - a_3c_2] - [b_2c_3 - b_3c_2][a_1c_3 - a_3c_1]} \tag{13}$$

$$\gamma_{12}^w(0) = \frac{-\left[a_3\sigma_y^2 + b_3\sigma_x^2\right]}{c_3} \tag{14}$$

where $a_1 = 1 - \varphi_{11}^2$, $b_1 = -\varphi_{12}^2$, $c_1 = -2\varphi_{11}\varphi_{12}$, $a_2 = -\varphi_{21}^2$, $b_2 = 1 - \varphi_{22}^2$, $c_2 = -2\varphi_{21}\varphi_{22}$, $a_3 = -\varphi_{11}\varphi_{21}$, $b_3 = -\varphi_{12}\varphi_{22}$ and $c_3 = 1 - [\varphi_{11}\varphi_{22} + \varphi_{12}\varphi_{21}]$.

Let Y_τ and X_τ be the *m*-period non-overlapping aggregates of y_t and x_t respectively. We now consider estimating the following bivariate VAR(1) from the temporally aggregated series:

$$\begin{pmatrix} Y_\tau \\ X_\tau \end{pmatrix} = \begin{pmatrix} \varphi_{11}^* & \varphi_{12}^* \\ \varphi_{21}^* & \varphi_{22}^* \end{pmatrix} \begin{pmatrix} Y_{\tau-1} \\ X_{\tau-1} \end{pmatrix} + \begin{pmatrix} E_{1\tau} \\ E_{2\tau} \end{pmatrix} \quad (15)$$

where $E_{i\tau}$ (*i=1,2*) represent the error process of the aggregated model. The OLS estimates $\hat{\varphi}_{ij}^*$ and $p\lim \hat{\varphi}_{ij}^*$ are given by:

$$\hat{\varphi}_{11}^* = \frac{\left(\sum Y_\tau Y_{\tau-1}\right)\left(\sum X_{\tau-1}^2\right) - \left(\sum Y_\tau X_{\tau-1}\right)\left(\sum Y_{\tau-1} X_{\tau-1}\right)}{\left(\sum Y_{\tau-1}^2\right)\left(\sum X_{\tau-1}^2\right) - \left(\sum Y_{\tau-1} X_{\tau-1}\right)^2} \quad (16)$$

$$p\lim \hat{\varphi}_{11}^* = \frac{\gamma_{11}^W(1)\gamma_{22}^W(0) - \gamma_{12}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2},$$

and similarly

$$p\lim \hat{\varphi}_{12}^* = \frac{\gamma_{12}^W(1)\gamma_{11}^W(0) - \gamma_{11}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2} \quad (17)$$

$$p\lim \hat{\varphi}_{21}^* = \frac{\gamma_{21}^W(1)\gamma_{22}^W(0) - \gamma_{22}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2} \quad (18)$$

$$p\lim \hat{\varphi}_{22}^* = \frac{\gamma_{22}^W(1)\gamma_{11}^W(0) - \gamma_{21}^W(1)\gamma_{12}^W(0)}{\gamma_{11}^W(0)\gamma_{22}^W(0) - \left(\gamma_{12}^W(0)\right)^2}. \quad (19)$$

Case 1: No Granger Causality Between the Variables in the Disaggregated Form

Let $\varphi_{12} = \varphi_{21} = 0$ and with $\sigma_{12} = 0$ the two series are uncorrelated.

$\Rightarrow \gamma_{ij}^{w}(k) = 0$ for all k and $i \neq j$ $(i, j = 1,2)$,from (10), (11) and (14)

$\Rightarrow \gamma_{ij}^{W}(k) = 0$ for all k and $i \neq j$, from (2)

$\Rightarrow p\lim \varphi_{12}^{*} = p\lim \varphi_{21}^{*} = 0$, from (17) and (18)

Key findings:

- If the cross-covariances between the disaggregated series are zero then the cross-covariances between the aggregated series will also be zero.
- If there is no Granger causality between the disaggregated series then the Granger causality between the aggregated series will also be absent.

Case 2: Causality Between the Disaggregated Series is One-Sided

Let $\varphi_{12} = 0$ such that x_t does not Granger cause y_t.

The causal parameters of the aggregated process can be written as

$$p\lim \hat{\varphi}_{12}^{*} = \frac{\varphi_{11}\left(1 + \varphi_{11} + \varphi_{11}^{2} + \ldots \varphi_{11}^{m-1}\right)^{2} \sigma_{y}^{2}\left[\gamma_{11}^{W}(0)\left(\frac{\varphi_{11}\varphi_{21}}{1 - \varphi_{11}\varphi_{22}}\right) - \gamma_{12}^{W}(0)\right]}{\gamma_{11}^{W}(0)\gamma_{22}^{W}(0) - \left(\gamma_{12}^{W}(0)\right)^{2}} \tag{20}$$

and $p\lim \hat{\varphi}_{21}^{*}$ remains unchanged as in (18).

Key findings:

- when $\varphi_{11} = 0$, $\varphi_{12}^{*} = 0$, suggesting that if the one-sided causality runs from a white noise series to a stationary series in the disaggregated form then temporal aggregation will not produce a spurious feedback relationship.

- Similar inference does not apply when $\varphi_{22} = 0$, thus we set $\varphi_{22} = 0.5$ in order to produce results in terms of 3-dimentional graphs.
- From Figures 1a-1c and from Table 1
    - As *m* increases VAR(1) tends to become VAR(0). However, when φ_{11} reaches unity, we get a near cointegrated specification and as a result VAR(1) remains VAR(1) as *m* increases.
    - When both φ_{11} and φ_{21} are of the same sign the feedback effect created is negative and when they are of opposite signs this becomes positive.
    - The magnitude of the spurious feedback is large for large positive φ_{11}. Since large positive φ_{11} is more likely in practice, spurious feedback is very likely with temporally aggregated data.

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Figure 1 and Table 1

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Case 3: Granger Causality Between the Disaggregated Series is Bi-Directional

In this case both φ_{12} and φ_{21} are non-zero. For the ease of computation, we set $\varphi_{11} = 0$ and $\varphi_{22} = 0$. The required aggregated parameters $(\varphi_{12}^*, \varphi_{21}^*)$ are given in (17) and (18), where

$$\gamma_{11}^w(2k-1) = 0, \quad \gamma_{11}^w(2k) = (\varphi_{12}\varphi_{21})^k \sigma_y^2 \quad \forall k = 1,2,.... \tag{30}$$

$$\gamma_{22}^w(2k-1) = 0, \quad \gamma_{22}^w(2k) = (\varphi_{12}\varphi_{21})^k \sigma_x^2 \quad \forall k = 1,2,.... \tag{31}$$

$$\gamma_{12}^w(2k-1) = \varphi_{12}(\varphi_{12}\varphi_{21})^{k-1} \sigma_x^2, \quad \gamma_{12}^w(2k) = 0 \quad \forall k = 1,2,.... \tag{32}$$

$$\gamma_{21}^w(2k-1) = \varphi_{21}(\varphi_{12}\varphi_{21})^{k-1} \sigma_y^2, \quad \gamma_{21}^w(2k) = 0 \quad \forall k = 1,2,.... \tag{33}$$

and $\sigma_x^2 = \frac{1+\varphi_{21}^2}{1-\varphi_{12}^2\varphi_{21}^2}$, $\sigma_y^2 = \frac{1+\varphi_{12}^2}{1-\varphi_{12}^2\varphi_{21}^2}$ (34)

Key findings:

From Figures 2a-2c and Table 2,

- As in the one-way causal system above, even in the feedback case the VAR(1) tends to become VAR(0) as *m* increases.
- What is more disturbing though is that a positive φ_{12} may become negative φ_{12}^*. Furthermore, the magnitudes of $p\lim \hat{\varphi}_{12}^*$ are such that in practice it is quite possible to conclude that causality is one-way though it is bi-directional.

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Figure 2 and Table 2

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Case 4: Contemporaneous Correlation

- generalizes Ericsson et al. (2000) result for any *m*.
  - with $\varphi_{11} = \varphi_{22} = \sigma_{12} = 0$ the contemporaneous correlation between y_t and x_t is zero (i.e., $\gamma_{12}^w(0) = 0$).
  - From the contemporaneous regression relationship $Y_t = cX_t + u_t$ with aggregated data we get $\hat{c} = \frac{\sum Y_\tau X_\tau}{\sum X_\tau^2}$, and

$$p\lim\ \hat{c} = \frac{\gamma_{12}^W(0)}{\gamma_{11}^W(0)} \quad (35)$$

Key findings:

Figures 3a-3c and Table 3,

- As observed by Ericsson et al. (2000) for *m*=2, the contemporaneous regression coefficient (also the correlation) could take positive, negative or zero at any level of aggregation.

- If both φ_{12} and φ_{21} are positive (negative) then the contemporaneous correlation will also be positive (negative). However, when the above parameters are of opposite signs then the sign of the contemporaneous correlation is determined by the sign of the larger of the two in absolute value.

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Figure 3 and Table 3

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