

Mathematics, anxiety, and the brain

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Abstract: Given that achievement in learning mathematics at school correlates with work and social achievements, it is important to understand the cognitive processes underlying abilities to learn mathematics efficiently as well as reasons underlying the occurrence of mathematics anxiety (i.e. feelings of tension and fear upon facing mathematical problems or numbers) among certain individuals. Over the last two decades, many studies have shown that learning mathematical and numerical concepts relies on many cognitive processes, including working memory, spatial skills, and linguistic abilities. In this review, we discuss the relationship between mathematical learning and cognitive processes as well as the neural substrates underlying successful mathematical learning and problem solving. More importantly, we also discuss the relationship between these cognitive processes, mathematics anxiety, and mathematics learning disabilities (dyscalculia). Our review shows that mathematical cognition relies on a complex brain network, and dysfunction to different segments of this network leads to varying manifestations of mathematical learning disabilities.

Keywords: cognition; math anxiety; math education; math learning disability; neural studies; parietal cortex; prefrontal cortex; working memory.

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Introduction

Mathematics is an integral developmental skill to attain. However, the processes and cognitive mechanisms to learn numeracy, arithmetic, and mathematics are complex. There is a complex interaction between executive functioning, working memory (WM), and long-term memory, to process and facilitate the necessary verbal, visual, and spatial information needed to solve simple and complex mathematical problems. In turn, if children or adults suffer from mathematical learning difficulties, measured by low mathematics achievement, this can lead to anxiety when individuals are faced with mathematical problems. Mathematical anxiety and low achievement can result in reduced performance at work, social, mental, and even legal domains (Gross et al., 2009). In this review, we discuss the relationship between performance on mathematical domains and cognition, mathematics anxiety, and mathematics learning disability. Understanding cognitive processes that enable efficient mathematical performance and decreasing anxiety is imperative for improving mathematics education. This can be done by conducting cognitive training methods that target processes underlying low mathematics achievement and mathematics anxiety.

Mathematics and cognition

There is a strong relationship between numerical cognition and executive functioning (De Stefano and LeFevre, 2004; Kolkman et al., 2013; Cragg and Gilmore, 2014), including WM, spatial processes, and linguistic skills. Executive functioning is important to encode, maintain, and manipulate information; these mechanisms are for numerous cognitive skills, including mathematics. Cragg and Gilmore (2014) provide an extensive review of the current literature within executive functioning and mathematics. Their review points toward the importance of WM, the ability to inhibit distractors and discern pertinent information, and to critically think, as predictors of mathematical aptitude. WM, language, and spatial skills are discussed below to highlight their role in mathematics, arithmetic, and numeracy.

Working memory

WM is a limited capacity storage system used for the manipulation and maintenance of information over short periods of time (Baddeley, 2003). Baddeley (2003) extended his original model of WM (e.g. Baddeley and Hitch, 1974; Baddeley, 2000) by providing neuropsychological evidence to support the neurological plausibility of the multicomponent model of WM. The multicomponent model suggests that WM is governed by a central executive, which selectively allocates cognitive resources to three subsystems: the phonological loop, visuospatial sketchpad, and an episodic buffer.

Central executive

The central executive is the attentional control system of WM, responsible for the allocation of attention across the different subsystems. The central executive is considered to be the most integral component of the model, as it is capable directing attention when our habitual responses are insufficient (Baddeley, 2003). Evidence has suggested that the frontal lobe is implicated with the functions of the central executive (Baddeley, 2003). The central executive's ability to control attention is essential for simultaneous processing within different WM subsystems, and the integration of information within the episodic buffer.

Phonological loop

The phonological loop is limited in capacity and is responsible for processing verbal information (e.g. numbers, letters, words, sentences, sounds, writing, and reading). This has been demonstrated through numerous studies investigating serial order effects (Burgess and Hitch, 1999; Hurlstone et al., 2014), word length effects (Cowan et al., 2003; Bhatarah et al., 2009), irrelevant sound effects (Chein and Fiez, 2010), language development (Baddeley et al., 1998; Baddeley, 2001), and language production (Kellogg et al., 2007; Acheson and MacDonald, 2009). In addition to behavioral studies, neuroimaging techniques have identified that the left ventrolateral prefrontal cortex becomes activated when processing and maintaining verbal information (Osaka et al., 2007). Postle (2006) also suggested that verbal WM storage typically activates the left posterior perisylvian areas. Taken together, there is behavioral and neurological evidence to support the existence of a specialized phonological construct responsible for processing and maintaining verbal information.

Visuo-spatial sketchpad

The visuo-spatial sketchpad is limited in capacity and is responsible for processing visual information such as color and shape, and spatial information such as location. The visuo-spatial sketchpad has been demonstrated through studies investigating change blindness (Baddeley, 2003), visual patterns tests (Pickering, 2001; Holmes et al., 2008; Quinn, 2008), mental rotation (Hyun and Luck, 2007), mental imagery (Logie, 1995, 2011; Neath and Surprenant, 2003; Chen and Cowan, 2009), and sequential movements (Willingham, 1998; Fiehler et al., 2008). However, in more recent years neurological evidence has suggested that a fractionation of the visual and spatial information is necessary, as they are two distinct processes (Klauer and Zhao, 2004; Holmes et al., 2008; Baddeley, 2012). For example, Postle (2006) pointed toward evidence suggesting that spatial WM typically activates parietal regions, caudate nucleus (motor planning), and extrastriate and parietal cortex regions (location). However, WM for objects was associated with activation within the ventral temporal and occipital cortex (geometric shapes), and posterior fusiform gyrus (faces). There is behavioral evidence to support the existence of a temporary store for visual and spatial information. However, neurological evidence suggests that these are two distinct processes with different neurological pathways.

Episodic buffer

The final component is the episodic buffer; the episodic buffer is controlled by the central executive and is multicode. It is limited in capacity but is able to integrate information from the phonological loop, visuo-spatial sketchpad, and long-term memory. The episodic buffer also addresses a longstanding criticism of the original multicomponent model, whereby the model did not explain the binding and chunking of multicode information. Rudner and Rönnerberg (2008) reviewed their previous research (e.g. Rudner and Rönnerberg, 2006; Rudner et al., 2007a,b) by discussing the neural underpinning of the episodic buffer during language processing. When bilingual participants were required to bind phonological representations of lexical items to their semantic meanings there was significant activation within the posterior temporal lobe and hippocampus. This is one of the first instances of the hippocampus being implicated with WM, but provides evidence to suggest that the episodic buffer utilizes long-term memory stores for encoding purposes.

WM is a complex cognitive construct that is capable of integrating and manipulating multiple pieces of sensory information in order to perform complex verbal, spatial, and visual tasks. In particular, this review will focus on the role of WM in mathematics performance. Mathematics is cognitively demanding and relies on WM in order to draw upon previously learnt information from long-term memory, and encode, store, manipulate, and solve problems that require an integration of multiple sensory stores to process. For example, the verbal components of mathematics (e.g. arithmetic and numbers) and the visual components of mathematics (e.g. geometry and diagrams; Raghobar et al., 2010), and long-term memory (previously learnt information).

Many studies have found a relationship between WM and math performance (Raghobar et al. 2010; Menon, 2014). For example, mental arithmetic requires multiple stages of processing, storage, retrieval, and integration. This includes holding partial results in memory when calculating equations with multiple steps (e.g. $20 \times 15 \div 6$). This process requires WM to adequately complete the task (Witt, 2011). There is evidence supporting the link between visuo-spatial WM and phonological WM to mathematical performance. Both subsystems play an important role in children's mathematical learning. The phonological loop has been implicated with mathematical performance in younger children (Wilson and Swanson, 2001; Swanson and Sachse-Lee, 2001; Swanson and Beebe-Frankenberger, 2004; Lefevre et al., 2010), while the visuo-spatial sketchpad has been found to predict mathematical performance in older children (Holmes et al., 2008; Lefevre et al., 2010). Lefevre et al. (2010) suggest that this might indicate that visual and spatial measures of WM may better predict mathematical performance than phonological measures in younger children.

WM has been found to predict mathematical skills among children and adolescents (Packiam Alloway et al., 2010; Witt, 2011). Witt (2011) investigated the effects of WM training on mathematical skills in children. The WM training which targeted inhibiting irrelevant information, monitoring, storage, and processing significantly improved mathematical skills compared to a control group. Witt's (2011) finding possibly explains the relationship between WM and mathematical cognition. There is also evidence to suggest that WM span (a measure of the limits of WM capacity) is also a determinant in predicting math performance in children.

Barrouillet and Lépine (2005) used WM span tasks (e.g. counting span and reading span) to identify if children (third and fourth graders) with higher WM capacities could solve simple mathematical equations quicker and

more accurately than children with low WM capacities. As predicted, children with high WM spans produced significantly more correct responses than the low WM span group. The time taken to respond was also correlated with WM span; as WM span increased, response times became faster. Further analyses confirmed that children with high WM capacities responded significantly faster than low WM capacity children. The authors concluded that WM span is an important factor in solving simple mathematical problems. Other studies have shown a similar effect, whereby WM performance contributed to mathematical achievement in second and third graders (Meyer et al., 2010). It appears that WM training improves executive functioning; this improves the mechanisms that are required to store, process, integrate, and retrieve information. In turn, these mechanisms enable children to improve mathematical performance by retrieving information from long-term memory to aid in solving math problems accurately and efficiently.

Neurological evidence suggests that WM training using mental calculation affected gray matter in the parietal and frontal cortex (Takeuchi et al., 2010). Specifically, Takeuchi et al. (2010) found that an individual's WM capacity is associated with the integrity of white matter. Furthermore, Takeuchi et al. (2010) found that the amount of WM training correlated with increased fractional anisotropy in the white matter regions. It is, however, not known which WM processes relate to math performance: duration of maintenance of information in WM, amount of information to be maintained in WM, and/or manipulation of WM information. These may be investigated using controlled WM tasks that manipulate WM duration and capacity and is an area that needs to be examined in the future (for these tasks, see Moustafa et al., 2013).

Along these lines, studies have found that executive functioning is closely related to children's mathematical skills (Bull and Scerif, 2001). Bull and Scerif (2001) investigated whether there was a correlation between executive functioning and mathematical ability in children. The study employed three different tasks: (1) Wisconsin Card Sorting Task (WCST)–revised (Heaton et al., 1993), which required that the child was to sort cards into groups based on color, shapes, and numbers; the criterion was altered without explicitly telling the child. The child's task was to adjust to the new condition; (2) Stroop task, where, for example, the children were asked to name the color which was red but the word was printed in green ink; and (3) Dual task performance (Baddeley et al., 1997): this task was designed to use both the articulatory loop and the visuo-spatial sketch pad and then compared to

a single task performance. The study found that children with lower mathematical abilities had difficulty with the Stroop task due to interference. Also, children with lower mathematical ability had problems adjusting and changing their strategy in the WCST. However, no significant correlation was found in the dual task condition, and the authors suggest that perhaps it is not a valid measure of executive functioning. Taken together these results indicate a clear correlation between executive functioning and mathematical abilities.

Spatial processes

Spatial skills typically refer to the ability to process and maintain spatial sequences involved in the planning of movements, visual representation of objects, routes, locations, and layouts of areas (Passolunghi and Mammarella, 2010). In addition to WM, studies also show that spatial skills are related to mathematical abilities. Studies have supported this finding by showing that spatial and visual WM abilities aid in arithmetic and mathematical problem solving (Reuhkala, 2001; Raghubar et al., 2010). This suggests that math ability is not just associated with verbal capacity but may also depend on visual and spatial ability. This has been confirmed by suggestions that preverbal children have a non-verbal representation of numerosity (Gallistel and Gelman, 2005). Passolunghi and Mammarella (2010) showed that participants who demonstrated deficiencies in spatial performance on the Corsi block tapping tasks also performed significantly worse in solving arithmetic problems.

There is a strong link between spatial processing and mathematics. For example, this has been demonstrated multiple times through the spatial numerical association of response codes effect (de Hevia et al., 2008; Hawes et al., 2015). This refers to the tendency to classify smaller numbers with ‘left side’ and larger numbers with ‘right side’. This was confirmed by Dehaene et al. (1993), who found participants responded faster with their left hand for smaller numbers and faster with their right hand for larger numbers. This indicates that behaviorally there is a spatial representation of numerical input, which categorizes numbers within with a visual and spatial domain, through mental imagery, and visual representations. There is also neurological support for the visual and spatial representation of numbers, arithmetic, and mathematics.

de Hevia et al. (2008) reviewed the role of visuo-spatial processing and identified multiple neuroimaging techniques showing that the posterior parietal areas are

activated when processing numerical or spatial information. Specifically, there is activation in the posterior parietal areas when processing simple multiplication, subtraction, and numerical tasks. Another notable area of activation was the horizontal segment of the intraparietal sulcus, which is typically activated when processing visuo-spatial information. However, in more complex mathematical problems neurological evidence suggests that a broader array of neural regions is activated. For example, pre-frontal regions are activated – which are typically involved in attention control, maintenance, and visuo-spatial WM processing (Postle, 2006; de Hevia et al., 2008); the inferior temporal gyrus is activated during mental imagery tasks, and the angular gyrus of the inferior parietal cortex is especially important for processing complex calculations when problems are presented visually. Taken together, there is a complex neurological network that is activated to process visual and spatial information as well as numerical information. Converging data suggest that there is an overlap between these two tasks, which typically activates the horizontal segment of the intraparietal sulcus and the posterior parietal areas.

Moreover, it was found that spatial skills relate to improved mathematical performance (Delgado and Prieto, 2004; Verdine et al., 2014; Zhang, 2014). Laski et al. (2013) investigated whether counting strategies have a negative impact on math achievement. The aim of the study was to examine the relationship between girls’ (first grade) verbal and spatial skills and how they solve arithmetic problems. Results suggested that girls with poor spatial skills are less likely to use higher-level mental strategies (e.g. direct retrieval or decomposition) for solving arithmetic problems (Casey et al., 2014). The same may perhaps apply to male students, although this was not investigated in this study. Studies also suggest that impairments in visual-spatial processing and WM in twins might explain the failure to develop numerical skills (Davidse et al., 2014). Research suggests that there is a gender difference in relation to spatial abilities (Kimura, 2000; Coluccia and Louse, 2004; Halpern, 2007). Tzuriel and Egozi (2010) examined whether intervention strategies could be used to decrease the effect of gender in spatial abilities. The sample was composed of 116 children who were 6 years old (58 boys and 58 girls). The children were divided into two groups, a control group and a group that was exposed to the intervention strategy. The intervention strategy was based on Wheatley’s (1996) ‘Quick draw’ activity that was designed to promote spatial sense in the math curriculum. Children in the intervention group were not trained to take a figure and mentally rotate the figure but rather perceive

the figure from different angles and retain it in their WM. In addition, the children were trained to conceptualize the figure as representing different objects, thus promoting flexibility. The study found that the experimental group's spatial ability improved significantly more than the control group. Within the experimental group there was a significant improvement in the spatial ability of young children and a mitigation of the gender gap between boys and girls. It is suggested that the same methods can help increase math skills, although to our knowledge, this was not previously done.

Linguistic skills

In terms of mathematics, linguistic skills enable conceptual cognitive representation which allows children and adults to understand and solve mathematical problems (e.g. 16 divided by 4 is equal to...; Desoete and Roeyers, 2005). The ability to understand the semantics of words such as 'divided', 'multiplied', 'less', and 'plus' is essential to calculate math problems that require language. If children's lexicons do not include these words or their semantic meaning, they may have trouble translating the words into mathematical equations. This notion has been supported by studies that have investigated the relationship between mathematical cognition and verbal/language processes. The study of Yackel et al. (1991) examined the effects of replacing standard math instructions (that are performed in front of the class by the teacher) with small groups within a class participating in problem-solving activities moderated by the teacher. They discovered that the group activities offered valuable opportunities for children to collaborate with one another and construct solutions for themselves verbally through language (talk) which was lacking with conventional instructions. Yackel et al. emphasize the importance of language to encourage collaborative learning, which enhances the semantic understanding of mathematic concepts. In this sense language facilitated learning which enhanced mathematical ability. However, due to the qualitative nature of the study, this concept needs to be further investigated to better understand what components of language facilitate mathematical learning (e.g. enhancing semantic meaning).

Mercer and Sams (2006) conducted a large-scale study similar to Yackel et al. (1991) where children were encouraged to participate by using explanatory language to describe their logic when solving math problems. The study revealed that children in the explanatory language group performed better in post-test assessments as opposed to the control group, indicating that language

can be used as a tool for developing children's ability to understand, reason, and problem solve math (Mercer and Sams, 2006). Importantly, it is also not known whether language understanding or speaking is associated with math performance.

Language knowledge (e.g. vocabulary as well as multilingualism) may be associated with strong representational processes of languages (including semantics and symbols) that may be associated with better math performance. Studies suggest a shared syntactic representation between numerical cognition and language (Scheepers and Sturt, 2014). Importantly, school motivation toward specific school subjects in the early elementary years specifically symbols and math equations plays a crucial role in language attainment and math performance (Guay et al., 2010), suggesting that the relationship between language and mathematical skills is bidirectional. Indeed, Scheepers and Sturt (2014) utilized a cross-domain structural priming methodology to investigate the bidirectional relationship between language and mathematics. Their stimuli consisted of mathematical equations and linguistic statements organized as either 'left-branching' (e.g. $2 \times 3 + 4$; I caught a ball) or 'right-branching' ($4 + 2 \times 3$; The ball I caught) statements. In the first experiment, mathematical equations were primed for language targets. In the second experiment language was primed for mathematical targets. Their findings identified a bidirectional relationship between language and mathematics, further emphasizing that language and mathematics share common syntactic representations. Future work should investigate which aspects of linguistic processes relate to better math performance.

Linguistic skills are important for solving math word problems, whereby written information must be transformed into numerical equations to solve a given problem. This process requires the ability to understand the semantics of the words, extract pertinent information, and compute correct equations (Hale et al., 2003). This process requires visual, spatial, and verbal WM to manipulate, transform, and calculate correct response. Dehaene et al. (1999) found that linguistic representations were important for exact arithmetic, which activated areas within the left inferior frontal circuit, which is typically observed in word association tasks (e.g. McCarthy et al., 1993). However, approximate arithmetic relies more on visual representation or non-verbal calculations, which requires visuo-spatial processing in the parietal lobes (Dehaene et al., 1999). Pica et al. (2004) investigated this notion further within an Amazonian group whose verbal system does not have numbers > 5 . Despite the limited lexicon for numbers, they were able to calculate approximate sums

beyond five items. However, they were unable to calculate sums > 5 for exact arithmetic. This confirms that there are differences in language-based arithmetic (exact arithmetic) and non-verbal arithmetic (approximate); this seems to be due to differences in the ability to visually represent items compared to using a limited verbal cache for arithmetic.

To sum up, the above studies have shown that numerical cognition relies on and is strongly interlinked with various processes, including WM, linguistic processes, and spatial abilities. These concepts are supported by LeFevre et al.'s (2010) comprehensive pathway model which was based on behavioral, cognitive, and neurological studies to determine three core predictors of mathematics proficiency. These were the ability to determine the quantities of numerical values, linguistic skills, and spatial skills. LeFevre et al. (2010) suggest that the model can be used to identify areas of mathematical deficiencies in children, for example, linguistic quantities or spatial deficiencies. However, further consideration should be given to understanding how spatial processing and linguistic processing may contribute to different types of arithmetic, approximate and exact, respectively.

We have outlined the complex nature of mathematics through behavioral studies and the underlying neural mechanisms of numerical, spatial, and verbal processing. Understanding these different processes is essential to teaching and aiding children with mathematical learning difficulties. By better understanding how math problems are processed and how spatial and linguistic processing can independently hinder math performance, teaching methods can be adjusted to target the problem areas. Targeting these areas might help reduce anxiety and negative feelings experienced by those with mathematical learning difficulties.

Mathematics anxiety

The term mathematical anxiety describes feelings of tension, apprehension, and fear upon facing mathematical content (Richardson and Suinn, 1972; Ashcraft, 2002; Fuller et al., 2016). The anxiety associated with mathematics can lead to avoidant behavior which results in impaired functioning for education, career opportunities, and day-to-day interactions (Dowker et al., 2016). Mathematics anxiety also appears to increase with age, specifically during childhood and in adolescents (Dowker et al. 2016). This was confirmed in a study by Blatchford (1996), who identified that attitudes toward mathematics

deteriorated significantly between children (11 years old) and adolescents (age 16 years). However, there is conflicting evidence in regard to when this decline in attitudes begins. Dowker et al. (2012) found no difference between third graders' (approximately 7.5 years of age) and fifth graders' (approximately 9 years of age) attitudes toward mathematics, despite evidence suggesting that differences in attitudes become more predictable in children in sixth grade compared to children in third grade (e.g. Gierl and Bisanz, 1995).

Children usually start their academic education with a motivated view of mathematics derived from their love for numbers (Ashcraft and Moore, 2012). However, a child may develop negative feelings and attitudes toward learning some subjects, such as mathematics (Dowker, 2005), which may cause anxiety when confronted with mathematical problems. It has been shown that fearful attitudes and negative perceptions of maths can have a long-term impact on career choices and development (Yackel et al., 1991). Furthermore, such negative attitudes toward mathematics were also found to impact behavior, communication, and many other social domains. Mathematical anxiety might be negatively related to an individual's ability to make advantageous choices and good decisions (Holmes and Gathercole, 2014; Morsanyi et al., 2014). Other studies have found an association between final secondary school math scores and math anxiety, which indicate that affective processes may directly be related to academic achievement (Galli et al., 2011). Achievement in mathematics-related tasks has been significantly correlated with attentional difficulties and social problems, but not with internalizing symptoms (such as anxiety, depression, and withdrawn behaviors; Wu et al., 2014). A relationship between math achievement and externalizing behavioral problems (includes behavioral and attention problems) has been shown to be stronger in girls compared to boys (Wu et al., 2014).

Math anxiety can impair daily activities such as calculating money when buying goods, understanding numerical information in large bills, and choosing investments based on statistics (Silk and Parrott, 2014). Studies investigating the relationship between math anxiety and educational level have reported significant correlations (Ashcraft and Ridley, 2005). However, math anxiety may not necessarily be associated with a lower level of math performance (Lyons and Beilock, 2012a,b). Laboratory investigations provide a better understanding of the negative consequences on cognition when math anxiety is present. For example, a higher number of errors and difficulties in avoiding incorrect answers are more pronounced with more complicated mathematical operations

in participants with higher math anxiety (Faust, 1996). Furthermore, participants with high math anxiety show rapid response and desire to end math tests quickly, and experience comfort upon ending their math test. Results also indicate that problem tasks including symbols are much more difficult than those containing simple numbers, and may also increase math anxiety (Ashcraft and Krause, 2007).

Math anxiety effects are evident in individuals with higher WM span whose performance is worse as a result of higher levels of anxiety (Mattarella-Micke et al., 2011). Evidence has consistently shown that anxiety reduces WM performance by impairing executive functioning and attentional control (e.g. inhibition and switching) which are essential for WM processes (Eysenck et al., 2005, 2007; Pnevmatikos and Trikkaliotis, 2013). In young children, math anxiety was shown to impair math performance for children with high WM compared to children with low WM (Ramirez et al., 2013). While this may appear counterintuitive, this occurs because children with high WM span rely on advanced strategies to solve mathematical problems, which require WM processes. Low WM span children rely less on WM processes as they do not utilize advanced strategies. As such, high-WM children are more susceptible to disruptions in WM functioning when experiencing math anxiety. Math anxiety has also been found to influence performance on a standardized math achievement test by impairing WM and acting as a resource-demanding second task (Ashcraft and Krause, 2007).

The Cognitive Reflection Test (CRT; Frederick, 2005) is a decision-making task involving the ability to suppress incorrect responses and avoid impulsive tendencies generated by math anxiety (Liberali et al., 2012). Cognitive reflection was found to be more closely related to a better performance on complex rather than on direct, simple calculation questions (Toplak et al., 2011). Recent studies postulate that low math grades and low scores on the CRT widely vary in individuals with math anxiety and that these individuals may also perform equally poorly in non-math-related cognitive tests (Frederick, 2005).

There is evidence that participants with math anxiety rely more on available heuristics, and more numbers within a small proximity of each other (Ramirez et al., 2013). Studies have also shown low levels of executive functioning in participants with math anxiety, either during challenging questions or task performance (Eysenck et al., 2007). WM can differentiate between participants' math performance; those having higher WM abilities may use availability heuristics significantly less than participants with lower WM abilities (Beilock and DeCaro, 2007). It may be that math anxiety impairs WM, which in turn

reduces the use of available heuristics; however, no study to date has investigated this link. Most studies regarding math anxiety and cognitive functioning have limitations in common: (a) they do not measure individual personality differences, for example, self-confidence; (b) they may be occasion or situation based; and (c) they do not consider cultural differences.

Mathematical learning disabilities (dyscalculia)

Children with mathematical learning disability (MLD) are often found to have disproportionately high levels of math anxiety (Carey et al., 2015). MLD is characterized by a discrepancy between performance on mathematical achievement tests and the expected performance based on age, intelligence, and years of education (American Psychiatric Association, 2013). For the purposes of this article MLD and dyscalculia are synonymous (Mazzocco et al., 2011). MLD is a cause of employment difficulties among adults (Butterworth et al., 2011). Mathematical deficits in children with MLD and their low math achievement peers include delays in learning mathematical procedures, understanding and representing the numerical magnitude, and impairment in retrieving basic arithmetic facts from long-term memory (Geary et al., 2012). About 6%–14% of school-age children have persistent difficulties learning mathematics but are attaining appropriate levels for all other domains (Barbarelli et al., 2005). In adults, MLD significantly interferes with daily activities (American Psychiatric Association, 2013). Hence, MLD can have lifespan consequences in many areas such as job attainment and achievement as well as social activities (Bynner and Parsons, 1997; McCloskey, 2007; Mazzocco et al., 2011).

MLD is one of the most common types of learning disabilities; however, it was found not to be related to intelligence as no systematic IQ difference was found between MLD children and their normally achieving peers (Shalev et al., 2005). Individuals with this disability have specific deficits in mathematical learning. Genetic factors may play a significant role in MLD and might be partially responsible for the comorbidity with other deficits such as reading disorders (Light and DeFries, 1995; Rousselle and Noël, 2007).

There are competing theories used to explain the underlying causes that maybe involved in MLD. First, the domain-general approach suggests that MLD results from a dysfunction in supporting cognitive systems such as

phonological skills, WM, long-term memory processing, and/or visuo-spatial processing (Geary, 2007; Mazzocco et al., 2011). In contrast, the domain-specific approach suggests the number sense theory in which arithmetic is dependent upon our ability to mentally represent and manipulate numbers on a ‘mental number line’ can explain MLD deficits.

Specifically, Dehaene (2001) suggests that deficit in the spatial system of number representation is the underlying cause of developmental MLD. The second alternate theory of Rousselle and Noël (2007) proposes that children with MLD have no difficulty processing numerosity but rather accessing numerical meanings from symbols, that is, mapping representation of numerical magnitude onto symbolic Arabic digits. The variation with differing theories may be explained by the mosaic nature of mathematics which involves expanding skills as the child develops and grows (Rousselle and Noël, 2007).

Importantly, many children with MLD have a reading disability or other cognitive difficulties that interfere with learning in school, such as attention deficit hyperactivity disorder (ADHD) (Barbarese et al., 2005; Fletcher, 2005; Shalev et al., 2005). In one study by Barbarese et al. (2005), between 57% and 64% of students with MLD also fulfilled the diagnostic criteria of a reading disability. In addition, many studies have reported that children with a specific learning disability often have social, behavioral, and emotional deficits (Swanson and Malone, 1992; Lopez et al., 1996). This suggests that attentional deficits and impulsivity may relate to impaired mathematical performance. Future research should investigate whether psychopharmacological and behavioral treatment for ADHD will ameliorate math performance deficits.

Neural substrates of mathematical cognition

There are few studies that have investigated the neural substrates of both mathematical cognition and mathematical anxiety. Menon (2014) provided an extensive review of arithmetic in adult and child brains and developed a neurocognitive model. The model was developed to understand the neural development of arithmetic; the basic components of the model suggest that (1) numerical formations and quantity representations are dependent on activation within the dorsal parietal and ventral temporal-occipital cortex; (2) WM is temporarily involved in the manipulation of quantities to form spatial and linguistic representations within the basal ganglia, dorsolateral

pre-frontal cortex, and frontoparietal regions; (3) the importance of long-term memory to engage with episodic and semantic memory to draw upon previously learnt information; this typically activates the medial and lateral temporal cortex; (4) executive functioning within the ventrolateral prefrontal cortex and anterior insula control attention and aid in problem solving.

Menon (2014) identifies that there are distinct differences in processing arithmetic in adult and child brains. These differences may be due to the automaticity of processing simple equations in the adult brain. This might be due to improvements in the ability to retrieve information from long-term memory for exact arithmetic. In turn, this allows more efficient processing due to a reduction in task difficulty associated with the angular gyrus. More recent evidence also suggested that the visual and spatial representation of numbers in the dorsal parietal and ventral temporal-occipital areas is activated when solving approximate arithmetic.

The triple code model attempts to describe the functional architecture and neural substrates of number processing (Dehaene et al., 1998). The model proposes three main representations of numbers: (1) a visual code, (2) a magnitude code, and (3) a verbal code.

However, more research is needed to map these to separable brain regions. Interestingly, Dehaene et al. (1999) found that humans use two systems for math cognition: spatial and linguistic, with exact mathematics reliant on linguistic processes, while approximate mathematics relies on spatial processing and the parietal cortex. The model accounts for numerical deficits, such as Gerstmann’s syndrome, which is a cognitive impairment that results from brain damage and can lead to dyscalculia among other symptoms. Gerstmann syndrome was found to be associated with damage the inferior parietal lobule (Carota et al., 2004; Vallar, 2007).

One study investigated whether brain operations related to numerosity predict acquisition of knowledge and academic achievement in mathematics (Haist et al., 2015). A magnetic resonance imaging (MRI) study (44 participants; 14 school-age children, 14 adolescents, 16 adults) examined non-symbolic task performance by comparing brain activity for numerosity precision with the Woodcock-Johnson III Broad Math index. Results suggested that the parietal cortex and intraparietal sulcus may lead to individual differences in achievement in mathematics. It is not known, however, whether these differences relate to exact or approximate mathematics, but given the Dehaene et al. (1999) findings, parietal activation differences may specifically relate to approximate math achievement and its underlying spatial processes. Moreover, a recent study

has investigated the representation of the numerical digit zero in the brain and found that the parietal cortex is active when monkeys represent zero in their brains (Okuyama et al., 2015). This perhaps suggests that we use spatial codes to represent the number zero.

Neural substrates of math anxiety

Few studies have investigated the neural substrates of mathematics anxiety. One study investigated the neurodevelopmental origins of math anxiety. The authors conducted a functional MRI (fMRI) study on young children (7–9 year olds; Young et al., 2012). Results indicated that math anxiety was associated with hyperactivity in the right amygdala regions, and with reduced activity in posterior parietal and dorsolateral prefrontal cortex regions. Also, children with math anxiety had elevated connectivity between the amygdala and ventromedial prefrontal cortex regions. These findings are in agreement with the role of the amygdala in anxiety disorders, such as generalized anxiety disorder (McClure et al., 2007; Monk et al., 2008), panic disorder (Sakai et al., 2005; van den Heuvel et al., 2005; Pillay et al., 2007), and social anxiety disorder (Tillfors et al., 2001; Blair et al., 2008; Evans et al., 2008; Guyer et al., 2008).

Using fMRI, Lyons and Beilock (2012a,b) investigated math anxiety by distinguishing between anticipation of doing math and ‘performance’ in math. Math anticipation was elicited by presenting a cue to identify the nature of the upcoming trials. For example, a flashing square was presented to identify that the upcoming trial was maths. Deficits in math performance were less severe for individuals with high math anxiety when they anticipated doing math, due to increased activity in frontoparietal regions. In addition, there was activity in the caudate, nucleus accumbens, and hippocampus during performance in the math task. According to the authors, math performance deficits are influenced by cognitive control before doing math and motivational resources during math performance.

There are neural substrates of math cognition and math anxiety. fMRI studies have investigated neural activity during anticipation of math-related tasks and during the task itself. During the anticipation stage, individuals with high math anxiety have exhibited increased activity of frontoparietal regions compared to those with low math anxiety. The frontoparietal regions involve the bilateral inferior frontal junctions, which are areas related to emotional control and re-experience of negative emotional

responses (White, 2009). During the completion of math-related tasks increased activity was exhibited in the caudate nucleus, nucleus accumbens, and hippocampus in individuals with high math anxiety compared to those with low math anxiety (Rudner et al., 2007a,b). These subcortical structures are involved in coordination of cognitive processing and motivation (Berlinger et al., 2008). Delayed and slow response in math-related tasks has been related to dopamine levels in the dorsal caudate and inferior frontal junction (Landau et al., 2009). Increases in dopamine have also been found to initiate mechanisms related to math-specific performance in individuals with high math anxiety before the actual math process starts (Haase et al., 2012).

Conclusions

Our review highlights the complexity of mathematical cognition, showing that it relies on various cognitive systems, including WM, spatial and linguistic mechanisms, as well as complex neural networks. Disorders affecting mathematics were found to be related to these cognitive and neural systems. For example, MLD is related to WM deficits, and less so to intelligence, as reported in studies discussed above. Recommendations for early interventions that focus on these cognitive deficits (such as WM training) are important as a main line of treatment. The National Mathematics Advisory Panel considered teacher-guided instruction based on the method of solving specific types of mathematics problems as the most effective intervention to highlight the importance of mathematics to everyday life problems (Gersten et al., 2008). Cognitive training methods may help improve concept formation, problem solving skill, memory, and eventually mathematical cognition (Sokolowski and Necka, 2016). Effective intervention methods must include several sessions ranging from weeks up to 6 months in order to reach noticeable improvements. These interventions must include the solving mathematical word problems, computational arithmetic problems, and novel word and arithmetic problems (Gersten et al., 2005).

As for math anxiety, current data highlight the importance of application of educational programs that combat the initial anxiety generated in individuals with high math anxiety rather than applying math courses trying to simplify math problems. Emotional control of responses related to math anxiety before it starts is argued to be a successful procedure to overcome math anxiety (Gresham, 2007). In addition, our review shows that it share some

similarities with other anxiety disorders. Therefore it is possible that treatments used for anxiety disorders can also be used for ameliorating math anxiety, including cue exposure therapy, which should help reduce anxiety when children are faced with mathematical problems.

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