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The Role of Survival Analysis in Financial Distress Prediction

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Abstract

Accurate business failure prediction models would be extremely valuable to many industry sectors, particularly in financial investment and lending. The potential value of such models has been recently emphasised by the extremely costly failure of high profile businesses in both Australia and overseas, such as HIH (Australia) and Enron (USA). Consequently, there has been a significant increase in interest in business failure prediction from both industry and academia.

Statistical business failure prediction models attempt to predict the failure or success of a business. Discriminant and logit analyses have been the most popular approaches, but there are also a large number of alternative techniques available. In this paper, a comparatively new technique known as survival analysis has been used for business failure prediction. In addition, hybrid models combining survival analysis with either discriminant analysis or logit analysis were trialled, but their empirical performance was poor. Overall, the results suggest that survival analysis techniques provide more information that can be used to further the understanding of the business failure process.

JEL Classification Codes: G33, G32

Introduction

The field of business failure prediction has many aliases, such as bankruptcy prediction, firm failure prediction and financial (de)stress prediction. Hereafter it will be referred to as business failure prediction (BFP). As the name suggests, BFP involves developing models that attempt to predict the financial failure of a business before it actually happens. Accurate BFP models would be extremely useful and valuable in the real world, as recently emphasised by the extremely costly failure of high profile businesses in both Australia (HIH and OneTel) and overseas (particularly Enron in the United States). Consequently, there has been a significant increase in interest in BFP, from both industry and academia.

Statistical BFP models attempt to predict the failure or success of a business based on publicly available information about that business, such as financial ratios from financial statements. In addition, some studies also include indicators of industry and economy wide performance to aid in the business failure predictions. The advantages of accurate business failure prediction models are that:

- Banks, investment banks, credit unions, and other financial institutions could avoid lending to businesses that will fail, and thus never repay their loans.

- The financial investment sector could improve the risk return trade-off from investments by not investing in failing businesses.
- Businesses could establish long-term relationships with other businesses (such as suppliers) that will not fail in the future, and thus increase the longevity and viability of their business relationships.
- Regulatory bodies, such as the Australian Securities and Investments Commission (ASIC), could make early identifications of failing businesses. This early identification would assist regulatory bodies in ensuring that business failure is 'handled' legally and illegal activities, such as avoiding taxes or diluting debt holders' claims by issuing substantial common stock dividends prior to failure, are avoided.

In addition to all the industry sectors that will profit from accurate BFP models, individuals and other entities dealing with businesses could also profit from using accurate BFP models in order to preferentially deal with successful businesses. Overall, accurate BFP models will increase people's confidence in investment, lending and the development of profitable business relationships, which will result in increased stable economic growth for the benefit of all involved.

The most important characteristics of a BFP model are its two forms of accuracy, namely classification and prediction. A model's classification accuracy is obtained by assessing its accuracy on the data set from which it was developed. Following that, the more important prediction accuracy of the model is assessed from its application to a brand new set of data, which reveals how well the model will perform on future predictions. Nevertheless, when measuring either classification or prediction accuracy there is a real world important consideration that should be noted. It is a more critical error to classify a failing business as successful (Type I Error) than to classify a successful business as failing (Type II Error). The reason for this is that a Type II Error only creates a lost opportunity cost from not dealing with a successful business, for example, missed potential investment gains. In contrast, a Type I Error results in a realised financial loss due to involvement with a business that will fail, for example, losing all money invested in an impending bankrupt business. Thus, the misclassification costs are not equal in the real world, and when analysing BFP models a higher weighted penalty should be imposed for a misclassification of a truly failing business (Type I Error). A quantifiable difference in misclassification costs has not been agreed upon in the literature, as it seems to vary for different circumstances and usually involves subjective decision making.

Additional information beyond the fail/succeed prediction is a desirable characteristic of a BFP model. As future predictions can never be made with absolute certainty, the confidence level (probability) of failure predictions is also useful. For example,

- Interest premiums on loans are usually based on the borrowing business' probability of failure.
- Regulatory bodies can focus their activities on businesses that have a higher probability of failure.
- Investors often weight a business' expected future cash flows at (1 – probability of failure) to more accurately calculate the fair price of a stock. This fair price can then be compared with the market price to determine whether to buy, sell or hold the stock.

Nevertheless, a model's accuracy still remains its most important characteristic.

The remainder of this paper is structured as follows. A brief review of BFP models is followed by an analysis and review of survival analysis for BFP. This is followed by sections explaining the data and methodology used, the results and the conclusions of the research. Gepp's thesis (2005) [available from http://www.it.bond.edu.au/publications/Theses.htm] contains more detail about each section; in particular, Chapter 2 contains a detailed review of BFP models and Chapter 4 details the research on BFP using SA techniques.

Business Failure Prediction Models: A Brief Review

Many different techniques have been applied to BFP since its beginnings in the 1960's. The field arguably started earlier, but the first statistical and mathematical models for BFP were published in the

1960s. Beaver (1966) presented a univariate model, then Altman (1968) pioneered the use of Multiple Discriminant Analysis (MDA) that was further developed by Deakin (1972), Edminster (1972) and others. Ohlson (1980) in his pioneer study, to avoid some significant problems associated with MDA, employed conditional Logit Analysis (LA) for predicting the survival of businesses. LA does not require normality nor equal covariances, which are pre-requisites for MDA. Subsequently both logit and probit models have been used with a focus of providing a measure of probability of business failure. Kumar and Ganesalingam (2001) have since focused on predicting the financial distress of a selection of major Australian companies. This research used principal component analysis, factor analysis, discriminant analysis and cluster analysis.

Based on Healy’s (1987) multivariate cumulative sum (CUSUM) method, Theodossiou (1993) introduced a sequential procedure to predict a business’ tendency towards failure. This procedure is based on the hypothesis that signals of a business’ deteriorating condition are produced sequentially for many years prior to failure. As the business’ economic condition deteriorates, its financial characteristics shift toward those of failed businesses and this procedure detects that shift. Theodossiou’s CUSUM procedures for BFP had excellent empirical results.

The soft computing methods known as artificial neural networks (ANNs) have also been used in BFP. Unlike traditional statistical techniques, ANNs do not require any restrictive assumptions such as linearity, normality and independence among input variables. These soft computing models are important as they offer qualitative methods that traditional quantitative tools in statistics and economics can not quantify due to the complexity of translating the systems into precise functions. ANNs have been shown to be good at classifying businesses into various groups based on financial distress. There are many research papers that apply ANNs to BFP, such as Odom and Sharda (1990) and Fletcher and Goss (1993) who respectively compared the performance of an ANN with a discriminant analysis and logit analysis model. More information about the various ANN methods applied in BFP are summarised in a book by Tan (2001).

There are also numerous other techniques that have been applied to BFP. For example, Wilcox (1976) applied the Gambler ruin model taken from probability theory to predict business risk and Casey (1980) used the human information processing (HIP) model to show that operating cash flow data can lead to more accurate predictions of business failure.

Survival Analysis (SA)

A survival analysis technique is the term applied to a dynamic statistical tool used to analyse the time till a certain event. Thus, the SA approach to BFP is fundamentally different from the other approaches mentioned above. While other techniques model BFP as a classification problem, SA models BFP as a timeline, where businesses are represented by lifetime distributions. Lifetime distributions are distributions with a nonnegative random variable that represents the lifetimes of individuals (or businesses) in some population. Lifetime distributions can be characterised by a number of descriptor functions, the most commonly being the survival or hazard function. The survival function S(t) represents the probability that a business will survive past a certain time t, while the hazard function h(t) represents the instantaneous rate of failure at a certain time t. The interpretations of these two functions is very different, but either one can be derived from the other.

There are many different SA techniques available to estimate the survival and hazard descriptor functions. These techniques use past data to calculate the functions at each specific time, but they do not have to ability to make future predictions. Thus, they can be used to analyse past failure to help further the understanding of the failure process. The most popular of these is a non-parametric technique known as the Product-Limit, or Kaplan-Meier, estimator. There is also a less-popular technique called the Nelson-Aalen Additive Estimator. This technique has some statistical advantages over the Kaplan-Meier estimator, which are briefly discussed by Harrell (2001) in Chapter 16. In addition to these techniques, there are also different SA models that define relationships between one of the descriptor functions (usually the survival or hazard function) and the set of explanatory variables. These models can also be used for prediction and are estimated using regression.

Regression Based Estimation

The basic difference between various SA models is the assumptions about the relationship between the hazard (or survival) function and the set (vector) of explanatory variables (*X*). Thus, the general regression formula can be written as $h(t) = g(t, X^T\beta)$, where X^T is the transpose of X, β is the vector of explanatory variable coefficients (also known as covariates) and g is an arbitrary function. In SA models estimated from regression it is customary to estimate the hazard rate, and then derive the survival rate as required. Traditionally, SA has been divided into two main types of regression models. These types are the proportional hazards (PH) and accelerated failure time (AFT) models, both of which have fully parametric and semi-parametric versions (refer to Prashanthi (2005) for more details). Due to its flexibility, the most prominent model applied in the medical and business failure field is the semi-parametric PH model defined by Cox (1972). Cox's PH model (Cox, 1972) is defined as $h(t) = h_o(t) \exp(X^T\beta+c)$, where:

- $h_o(t)$ is termed the baseline hazards function and describes how the hazard function changes over time and is the nonparametric part of the model; and,
- $\exp(X^T\beta+c)$ describes how the hazard function relates to the business specific explanatory variables and is the parametric part of the model, where c is an estimated constant. Note that some or all of the explanatory variables can be time dependent.

The regression coefficients β are calculated by an efficient method very similar to the maximum likelihood method (detailed in Kalbfleisch and Prentice (1980)). Furthermore, as with traditional regression techniques, the best explanatory variables are chosen from a starting set by forward or backward selection methods.

Theoretical Analysis

SA techniques are more sophisticated than the traditional popular techniques of discriminant analysis (DA) and logit analysis (LA). Except for sequential CUSUM procedures, SA is the only well-known technique that incorporates the time series (or longitudinal) nature of BFP data into its model. Thus, SA does not assume that the failure process remains stable over time. All other cross sectional models are only valid if the underlying failure process remains stable over time, which is a problem as the steady failure process assumption is usually violated in the real world (Laitinen and Luoma, 1991). This fundamental difference between the time-series SA models and cross-sectional traditional models also makes empirical comparisons between the techniques difficult. For example, a single SA model can make predictions of varying length; however, a single DA model can only make predictions of a fixed length based on its training data. Therefore, a single SA model is usually compared with many traditional models. This is an advantage in itself as one SA model is clearly more powerful in making different predictions than one traditional model.

The built in time factor in SA models allows them to model time dependent explanatory variables. Zavgren (1985) found that in BFP the signs of the explanatory variable coefficients may change in different years before failure. Laitinen and Luoma (1991) went further and added that the values of the coefficients may also change relative to time before failure. Thus, an advantage of SA is the capability to model these changes, which can not be done with cross sectional models. Therefore, SA models appear to be more suited to modelling a dynamic process, such as business failure, than cross sectional models. This also means that theoretically, the predictive accuracy of SA models should be greater than that of both DA and LA.

Almost all well-known approaches assume that the data (businesses) comes from two distinct populations, which are those either going to succeed or fail. SA models do not make this assumption, but rather assume that all businesses come from the same population distribution. In SA models, the successful businesses are distinguished by treating them as censored data, which indicates that their time of failure is not yet known. This assumption more accurately models the real world (Laitinen and Luoma, 1991). SA models can also deal with the delayed entry and early exit of businesses from a study, which is likely to happen in studies of business failure. Furthermore, SA does not make any of the restrictive distribution assumptions inherent in DA and LA, such as linearity. The semi-parametric and parametric SA models make some distribution assumptions, but they are less commonly violated.

In addition to the easily interpretable probability of success or failure, SA models also produce the interpretable hazard function that is not available in other techniques. Analysis of the hazard rate can aid understanding of any process of death or failure (Harrell, 2001). Thus, SA is able to provide more information than other techniques, which is a significant component of any good model (Chatfield, 1995).

There are also a few disadvantages associated with the use of SA. There is evidence to suggest that the sample construction, specifically the proportion of failing and successful businesses, may affect the estimation of the SA model. However, this problem seems to be minor as most randomly selected BFP data sets contain a mixture of failing and successful businesses. Some researchers also identified that SA techniques (particularly the Cox model) are subject to multicollinearity problems, but these can be easily avoided by using standard forward and backward variable selection procedures. A more important disadvantage is that SA is designed to focus on determining the effects of explanatory variables on the life of businesses, rather than being designed to predict outcomes such as the failure of businesses. The ramification of this is that obtaining predictions from SA models is more difficult than anticipated.

Review of Survival Analysis in BFP

SA has not yet become as popular in BFP as DA and LA, but it is considered to be a popular alternative to these main techniques. The pioneering paper on SA applied to BFP is by Lane et al. (1986), who used the Cox model to predict bank failure. Lane et al. created their model based on a selection of 334 successful and 130 failed banks from the period 1979 to 1983. The model was then tested on a hold-out sample with one and two year predictions, in which the cut-off value was set at the proportion of failed banks in the sample. The prediction accuracy of the Cox model was found to be comparable with DA on the initial and hold-out data, but the Cox model produced lower Type I Errors. In addition, Crapp and Stevenson (1987) applied a Cox model to some Australian credit unions with similar encouraging results.

Laitinen and Luoma (1991) again applied the Cox model to business failure. The significance of this paper is that it was the first to critically present the advantages and disadvantages of using SA to predict business failure. Laitinen and Luoma also empirically compared the classification accuracy of the Cox model with DA and LA using 36 failed Finnish limited companies and 36 successful counterparts. Their predictions were made by dividing the businesses into two groups based on their hazard ratios, according to the ratio of failed and successful businesses in the original sample (equal groups in this case). Businesses in the group with the higher and lower hazard ratios were then predicted to fail or succeed respectively. Although the techniques were comparable, DA and LA were found to be slightly superior predictors to the Cox model. Nevertheless, Laitinen and Luoma argued that the SA approach was more natural, appropriate and flexible, and used more information. It was also stated that the empirical underperformance could have been due to the small sample or sample bias inadvertently caused by the authors. Therefore, it was the author's belief that further research into SA as a BFP tool would result in SA models becoming superior to traditional models. Earlier support was also given by Keasey et al. (1990) and Ogg (1988) who recommended that SA techniques be used in BFP.

Kauffman and Wang (2001, 2003) used SA techniques to examine the drivers behind the survival of Internet businesses. The data set comprised quarterly data on 100 Internet businesses from the period of 1996 to 2001. Six explanatory variables were used: one industry specific, two business-specific, two ecommerce specific, and one macroeconomic variable. Two SA techniques were applied to this data: a Kaplan-Meier model was used to perform a descriptive analysis, and the Cox model was used to explore the relative strengths of explanatory variables. Useful conclusions, such as businesses targeting both commercial and consumers groups are less likely to fail, were drawn from both of the

SA techniques applied. Although this work did not develop an SA model for predicting business failure, it demonstrated the usefulness of SA techniques for researching the business failure process.

Shumway (2001) applied the first SA model to a data set of significant size. The model was formed using various financial ratios and market-driven variables for over 2000 companies from the NYSE and AMEX over 31 years. This was the pioneering use of a multiperiod logit model to estimate the SA model coefficients. This allowed Shumway to estimate an AFT SA model, which had not been previously applied to BFP. Consistent with previous studies, Shumway noted the theoretical superiority of SA techniques over the more popular techniques (DA and LA). In addition, Shumway's SA model was shown to empirically outperform both DA and LA in hold-out predictions. However, less than 10% of the businesses in the data set were failed, which is much lower than the percentage in the real world. In addition, Shumway only considered Type I Error.

Laitinen and Kankaanpää (1999) presented a comparative study, in which the Cox model along with DA, LA, RPA (a decision tree approach), ANN and HIP were analysed. The six techniques were empirically compared for their 1, 2 and 3 year prediction accuracy using a data set containing three explanatory variables from 76 Finnish companies (with equal number of success and failures). Their analysis showed that SA had superior predictive power for 2 and 3 year predictions. However, they concluded that there were no statistically significant differences in the predictive powers of any of the six models, except for LA being slightly superior to SA for one year predictions.

Overall, there have been few studies on the application of SA to BFP, and most of the previous research has used Cox's model. Although Lane et al. (1986), Laitinen and Luoma (1991) and many more have indicated that the Cox model was very appropriate for use in BFP, it has not been consistently shown to be superior to traditional techniques. Lane et al. (1986) found the Cox model to slightly empirically outperform DA, but Laitinen and Kankaanpää (1999) found no overall statistical difference between the empirical performance of DA and LA, while Laitinen and Luoma (1991) found both DA and LA empirically superior to SA. Therefore, it would be valuable research to apply the Cox model to a large set of data and compare it to both DA and LA again. Furthermore, comparing the techniques across different misclassification costs has never been done.

Data Analysis and Methodology

The main goal of this current research is to assess the empirical classification and prediction accuracy of the Cox SA model when applied to BFP. This analysis has been undertaken on a sufficiently large data set and over different years of predictions with different misclassification costs, in order to address the gaps in the SA literature identified above.

Data

A large time-series data set has been used for this research. This data set was acquired from Kahya and Theodossiou (1999) who have previously undertaken BFP studies. The main properties of this time-series data set are presented in Table 1. More information on the data, the list of explanatory variables and the definition of business failure used can be found in Kahya and Theodossiou's (1999) paper on the CUSUM procedure. It is important to note though that there is no sampling bias in the data, which is a weakness of the previous major BFP study of the Cox model by Laitinen and Luoma (1991).

Table 1: Description of Data Used.

Property	*Value*
Businesses (Type)	Manufacturing and Retail (from AMEX and NYSE)
Selection Procedure	Random selection
Businesses (Number)	189: 117 successful and 72 failed
Explanatory Variablest	27 financial variables: mostly financial ratios
Time-line	1974-1991 (18 years)
Number of Business-Years (Instances)	2,954

A separate hold-out data set was created for the purpose of estimating each model's ability to predict the failure or success of businesses not used in the model estimation process. The hold-out data set was created as approximately 10% of the size of the initial data set with similar proportions of successful and failed businesses. The 11 successful and 7 failed businesses were randomly chosen from the initial data set of 117 successful and 72 failed businesses; therefore, the training data set comprised 106 successful businesses and 65 failed businesses. Overall, this meant that the training data set comprised 2,669 instances, and the hold-out data set comprised 285 instances.

Methodology

Cox, DA and LA models were all estimated using forward stepwise regression procedures with the significance level boundaries for entry and removal set to 5% and 10%. The classification and prediction ability of the three models was then analysed and compared. The classification ability was determined by how accurately a model classified businesses in the original training data set, known as 'in-sample' classification. The prediction ability was determined by how accurately a model classified new businesses from the hold-out data set.

The different models were compared based on prediction intervals of 1-year, 2-year, and every subsequent yearly period for up to 10-years. The purpose of this extensive comparison was to reveal whether certain techniques perform better at shorter or longer term predictions. Due to the time-series nature of the Cox model, a single Cox model was compared with a separate DA and LA model for each different prediction length. It is arguable that this could bias the results towards the traditional models, but this is the process that would be undertaken for real-world predictions of different length with both DA and LA.

The different models have also been compared for various misclassification costs over each prediction length to assess how they adapt to higher Type I Error costs. All three techniques output a probability of survival, which is then compared with a cut-off value ranging between 0 and 1 to determine whether the prediction is for failure or success. For the SA model, the cut-off values were compared with values of the survival function. The different misclassification costs were achieved by varying the cut-off value for the probability of success. Usually this cut-off value is set to 0.5 representing equal misclassification costs, whereby a business with a probability of success/survival greater than 0.5 results in a successful prediction (else a fail prediction if less than 0.5). The DA, LA and Cox models have been compared over 8 different cut-off values, ranging from 0.5 up to 0.85, in 0.05 intervals. Higher cut-off values represent higher Type I Error costs. Thus, cut-off values lower than 0.5 were not studied as they represent Type II Error being more costly than Type I Error, which is not a realistic scenario.

To analyse whether a combination of the Cox model with one of the most popular techniques (DA or LA) would better predict business failure, two hybrid models were also developed. These hybrid models were generated by including the values of the survival function as an extra explanatory variable in the DA or LA model estimation process. The survival function values included in the data set matched the prediction length of the DA or LA model; for example, S(3) values were used for the DA and LA model for 3-year ahead predictions. The DA and LA models were then developed (and analysed) for each prediction length in the same way as described above, but with this extra explanatory variable.

Results

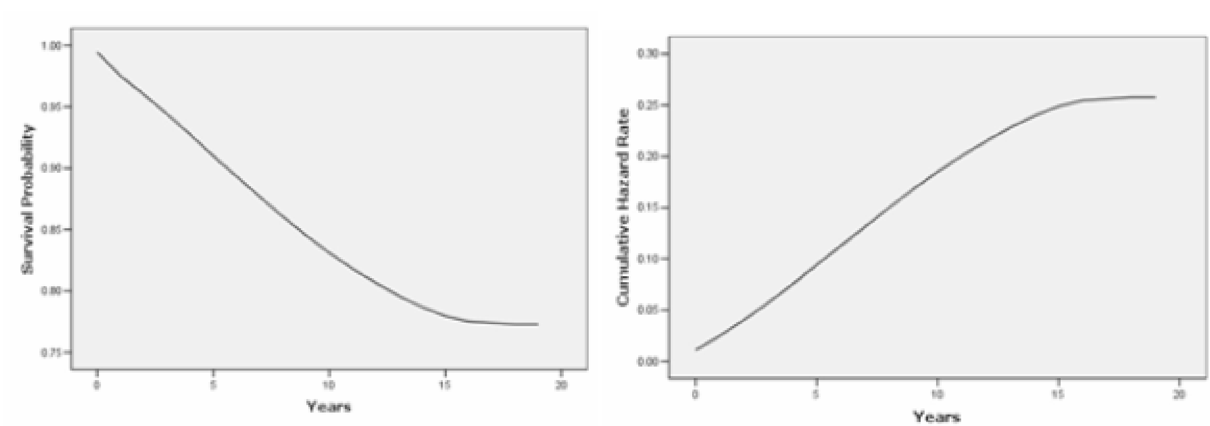
The results are structured as follows. Firstly, the change in the business failure process over time is analysed by using information obtained from the Cox model. The empirical performance of the techniques is then investigated. The DA, LA and Cox models are compared with equal misclassification cost, and with varying misclassification cost for both classification and prediction accuracy. The hybrid models are then analysed. Finally, the relative significance and importance of the explanatory variables is analysed. Summary tables of the classification and prediction accuracy of the 3 main models are presented in Appendix A and Appendix B respectively.

Business Failure Process

The Cox model automatically outputs the survival function and the cumulative hazard function, and information on how they change over time. The survival and cumulative hazard function for an average business over time is shown in Figure 1. This graph reveals a linear decline in the survival rate of a business over the first 15 years of it being included in the study. The survival probability of a business reduces by approximately 8% each year. However, after 15 years the survival probability does not drop significantly and remains steady at about 77%. A consistent trend was found from analysing the cumulative hazard function. The cumulative hazard function is shown to increase in a linear trend for the first 15 years, but then remains somewhat constant thereafter. This means that assuming a business has survived for 15 years or more, the likelihood it will fail in the near future is low. In addition, the probability of survival for more than 15 years is not significantly lower than the probability of survival for 15 years. This seems intuitively logical as established businesses have historically been considered much less likely to fail than relatively new businesses, although 15 years is longer than expected for a business to become established.

This analysis is an example of how the extra information automatically generated by the Cox model (as a SA technique) aids the understanding of the business failure process.

Figure 1: Survival (left) and Hazard (right) Function at the mean of explanatory variables.



Equal Misclassification Costs

The in-sample classification accuracy of the three models over the 10 different prediction lengths is presented in Figure 2. Overall, the performances of the LA, DA and Cox models are very similar with equal misclassification costs. It is also apparent from this graph that the classification ability of all techniques reduces as the prediction interval increases. All three models correctly classify about 96%

of businesses according to their possible failure within one year; however, this accuracy decreases constantly to approximately 82% accuracy when classifying businesses according to their possible failure within 10 years. Although the decline in classification accuracy occurs mostly in a linear trend, the graph suggests that the accuracy level may flatten out for longer prediction intervals, especially for the DA and LA techniques. Nevertheless, this decline in accuracy is not a large problem, as an 82% correct classification percentage indicates a good model that will be useful in the real world.

Figure 2: Percentage of correct in-sample classifications.

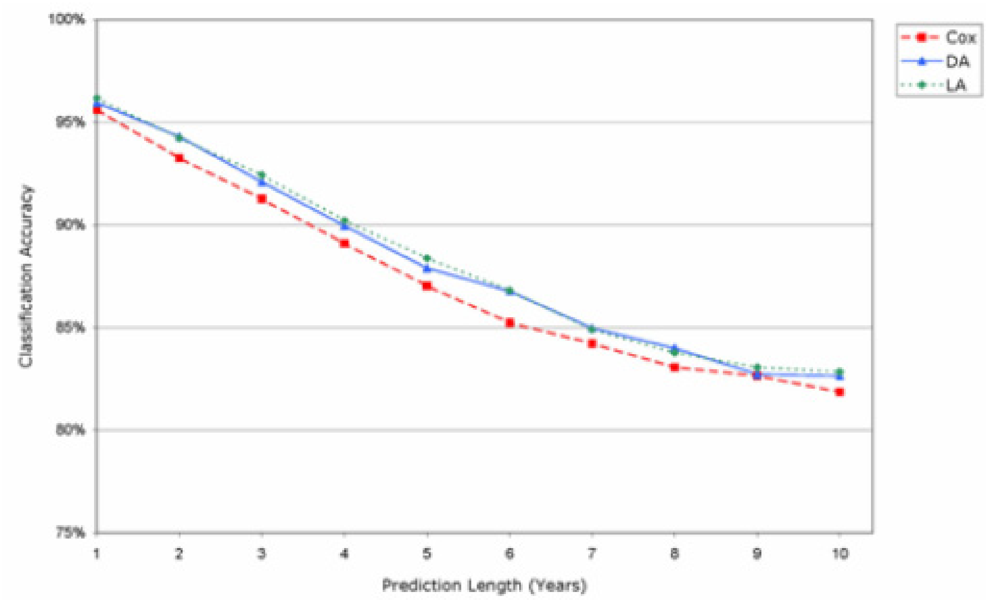
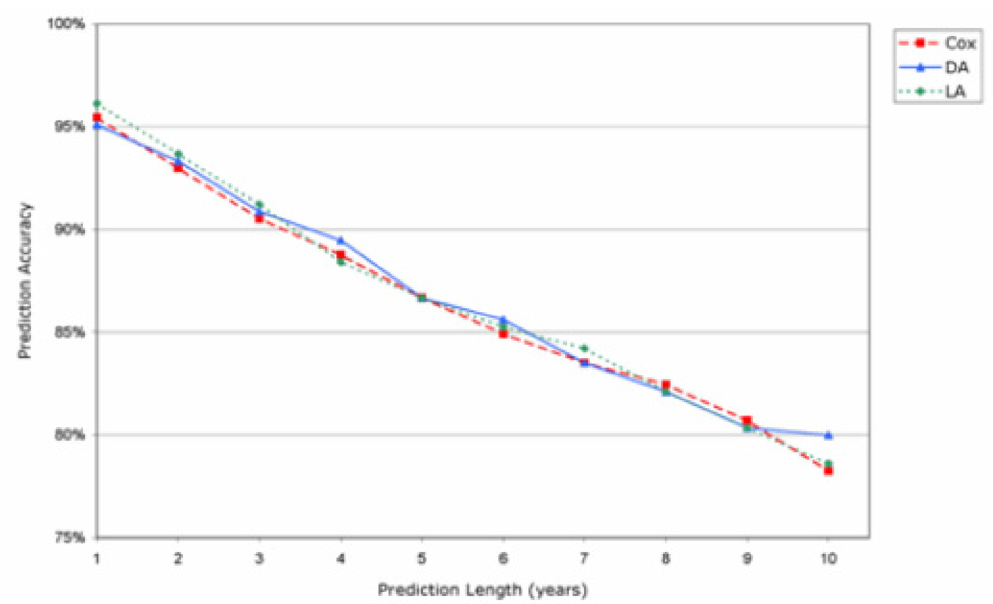


Figure 2 also shows that the DA and LA models always have a slightly higher number of correct classifications compared with the Cox model. However, the difference between the correct classification percentages is less than 1% on average, with the largest difference when classifying businesses based on their possible failure within 6 years: the DA model and LA model correctly classified 1.54% and 1.57% more businesses respectively. Although the DA and LA lines in Figure 2 are very similar, the LA models had a slightly higher correct classification percentage than the DA models for 6 out of the 10 different prediction intervals. This is consistent with previous DA and LA research that found the two techniques to be very similar in terms of classification ability, and that the LA models are more often slightly better. It is also interesting to note that the relative performance of the Cox model did not improve for longer prediction intervals (up to 10 years). On average over the various prediction intervals, the number of correct classifications made out of the 2,669 in-sample classifications was 2,356, 2,352, and 2,331 for the LA, DA and Cox model respectively. Hence, the Cox model is very similar to both DA and LA in terms of classification ability, but the relative order of the techniques according to classification ability with equal misclassification costs is LA, DA and then the Cox model.

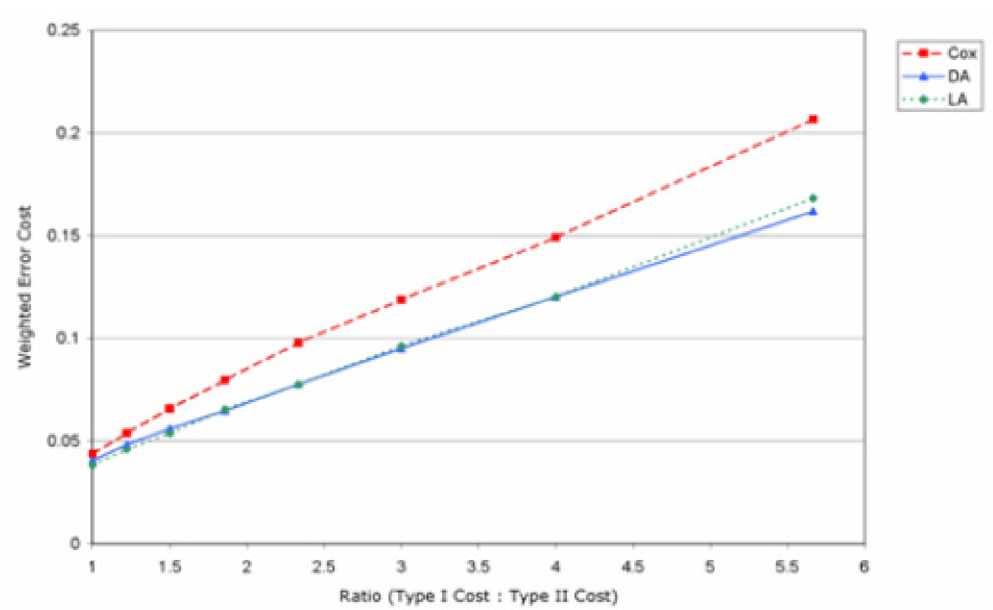
The prediction accuracy is more similar than the classification accuracy for equal misclassification costs, as shown in Figure 3. It is again evident that the accuracy of the models decreases as the prediction interval increases. This result is consistent with the intuitive thought that it is harder to classify and predict events that occur further into the future. From comparing this graph with the previous graph of classification accuracy (Figure 2) it can be seen that there has only been a small decline in accuracy when moving from the in-sample to hold-out data. This suggests that all the techniques have successfully generated models predominantly based on general trends in the training data set, and overly complex models have not been created. However, it is observable that the loss in accuracy between classification and prediction is more evident with longer prediction intervals.

Figure 3: Percentage of correct hold-out predictions.



When moving from classification to prediction, the LA, DA and Cox models respectively lost 1.62%, 1.43% and 0.91% accuracy on average. That is, the techniques declined in accuracy proportional to their in-sample classification accuracy. Hence, the predictive ability of all the three techniques is almost the same, and the best predictive technique depends upon the prediction interval. The Cox model is the best for 8 and 9 year ahead predictions, but DA is more than 1% more accurate than both the Cox and LA model for 10 year ahead predictions. Therefore, again there is no evidence to suggest that the Cox model performed better with longer prediction intervals. From 285 predictions made, the LA and DA techniques averaged 247 correct, while the Cox model averaged 246. Thus for equal misclassification costs, the Cox model has produced almost identical prediction accuracy compared with LA and DA models.

Figure 4: The cost of one year ahead in-sample classifications.



In-sample Classification

The in-sample classification accuracy over different misclassification costs was analysed for each prediction length. Due to the varying misclassification costs, a weighted error cost measure was used to compare the classification (and prediction in the following sub-section) accuracy of the techniques. This cost measure weighted the number of Type I and Type II Errors by their respective relative costs, which enabled unbiased comparisons to be made that allowed for the effect of the varying misclassification costs.

The comparison of the costs of the Cox, DA and LA models for classification based on failure or success in one year is presented in Figure 4. The Cox model is clearly the worst performing technique with the highest weighted error cost for all misclassification costs. In addition, the Cox model does not adapt as well as the other techniques to increasing Type I Error costs. Therefore, the relative superiority of DA and LA as classifiers increases as the cost of Type I Error increases. The DA and LA models have very similar performance, where LA was marginally better for lower Type I Error costs, but DA was marginally better for the highest Type I Error costs.

Averaged over the various prediction intervals and misclassification costs, the classification accuracy of LA and DA were very similar, but LA was slightly superior. The observation that the Cox model was more costly than DA and LA, especially for high Type I Error costs, continues through to prediction intervals of ten years. However, the performance gap between the Cox model and DA and LA models decreases as the prediction interval increases from one year to six years. In addition, the increase in the performance gap for higher Type I Error costs becomes less significant as the prediction interval increases. These observations are illustrated in Figure 5 and 6, which shows the results for prediction intervals of 3, 6 and 10 years. Thus, the relative superiority of the DA and LA models over the Cox model decreases as the prediction interval increases. In addition, the Cox model adapts better to higher Type I Error costs as the prediction interval increases.

Figure 5: The cost of three year ahead in-sample classifications.

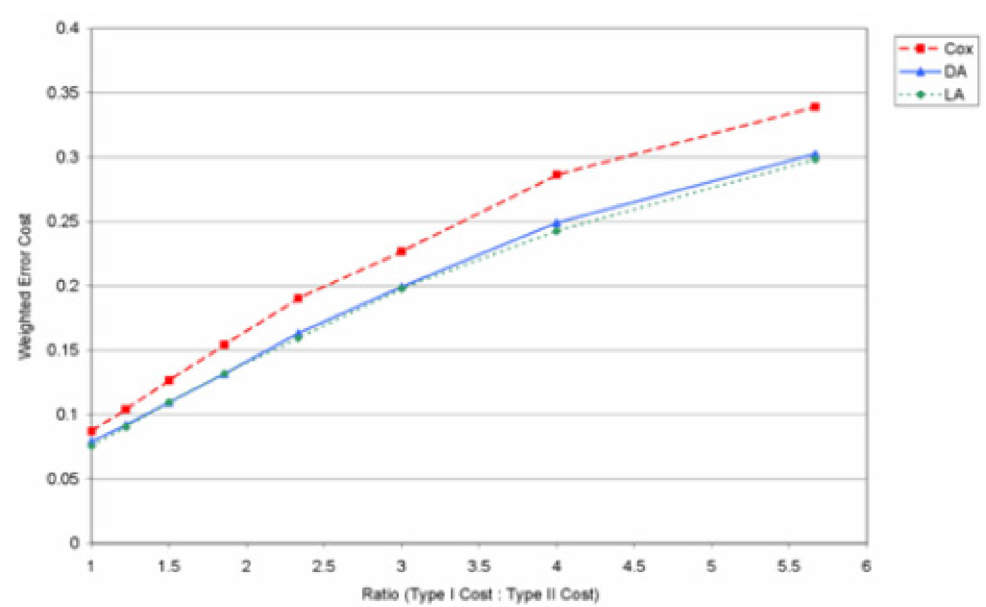
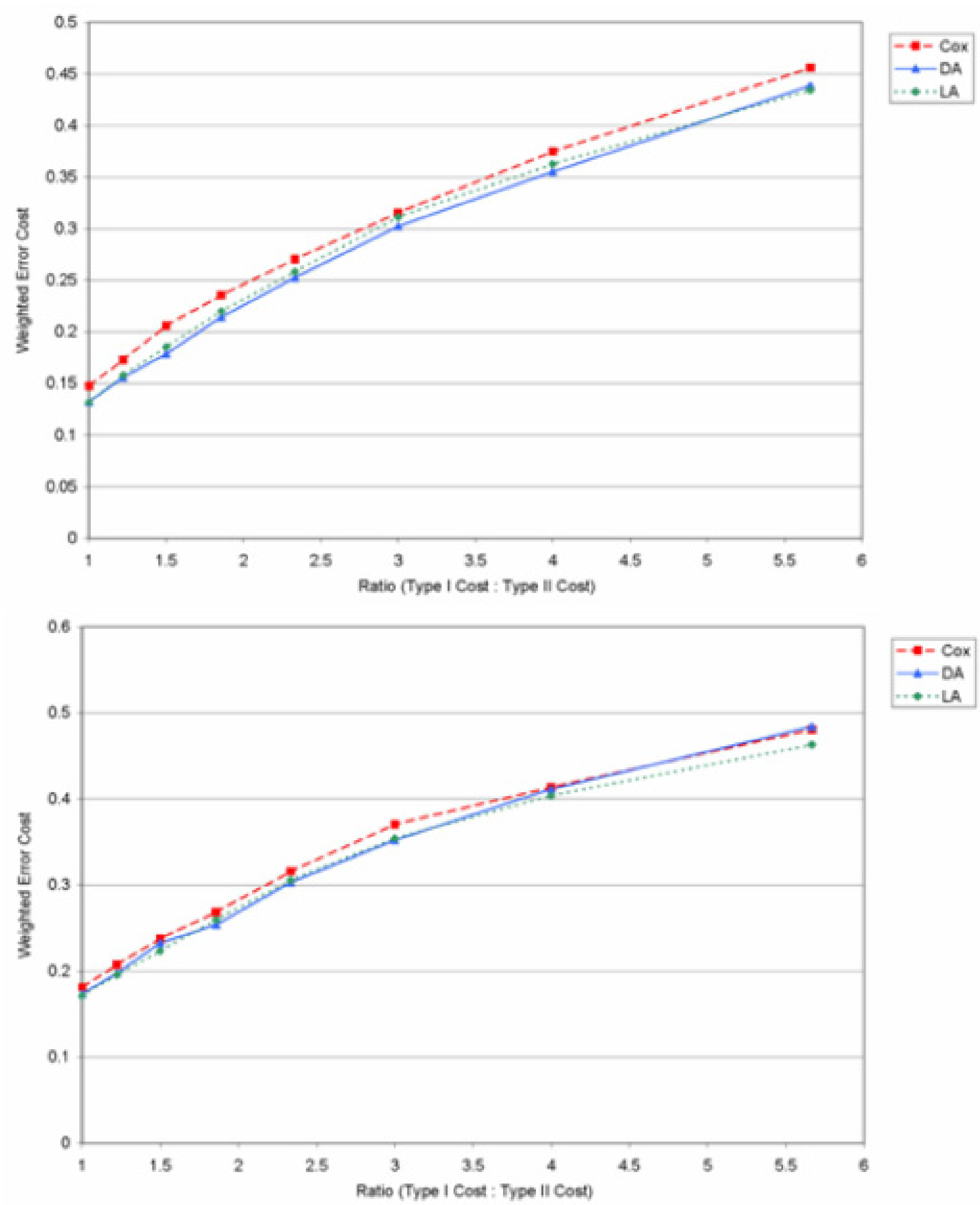


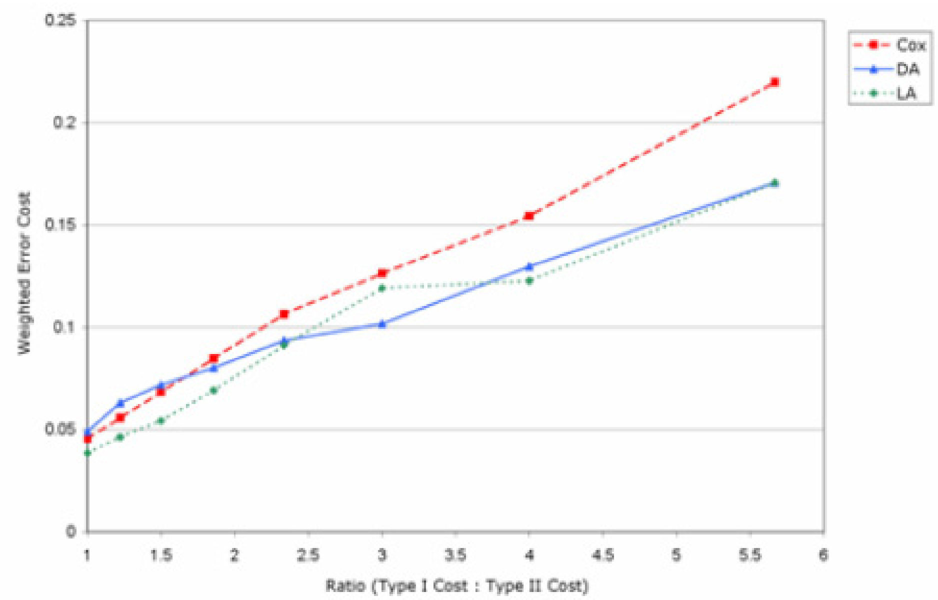
Figure 6: The cost of six (top) and ten (bottom) year ahead classifications.



Hold-out Predictions

The prediction accuracy was more varied than the classification accuracy discussed in the above sub-section. The one year prediction accuracy, estimated on the holdout data, for the Cox, DA and LA models is presented in Figure 7. Although the Cox model, had a lower cost compared with the DA model for lower Type I Error costs, the Cox model did not adapt well to larger costs of Type I Error. Thus, the Cox model became significantly more costly than its DA and LA model alternatives for higher Type I Error costs. The best predictor between LA and DA was LA for all but one misclassification cost.

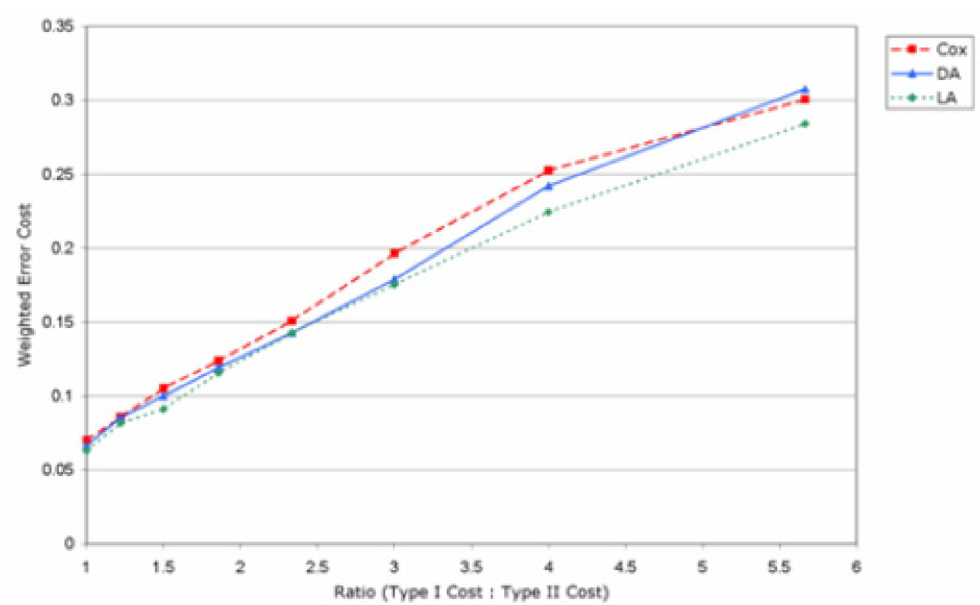
Figure 7: The cost of one year ahead hold-out predictions.



Prediction intervals from two to ten years reveal a general trend of higher costs for Cox models that become worse with higher Type I Error cost. However, there are some slight variations in these general trends that are explained and illustrated below.

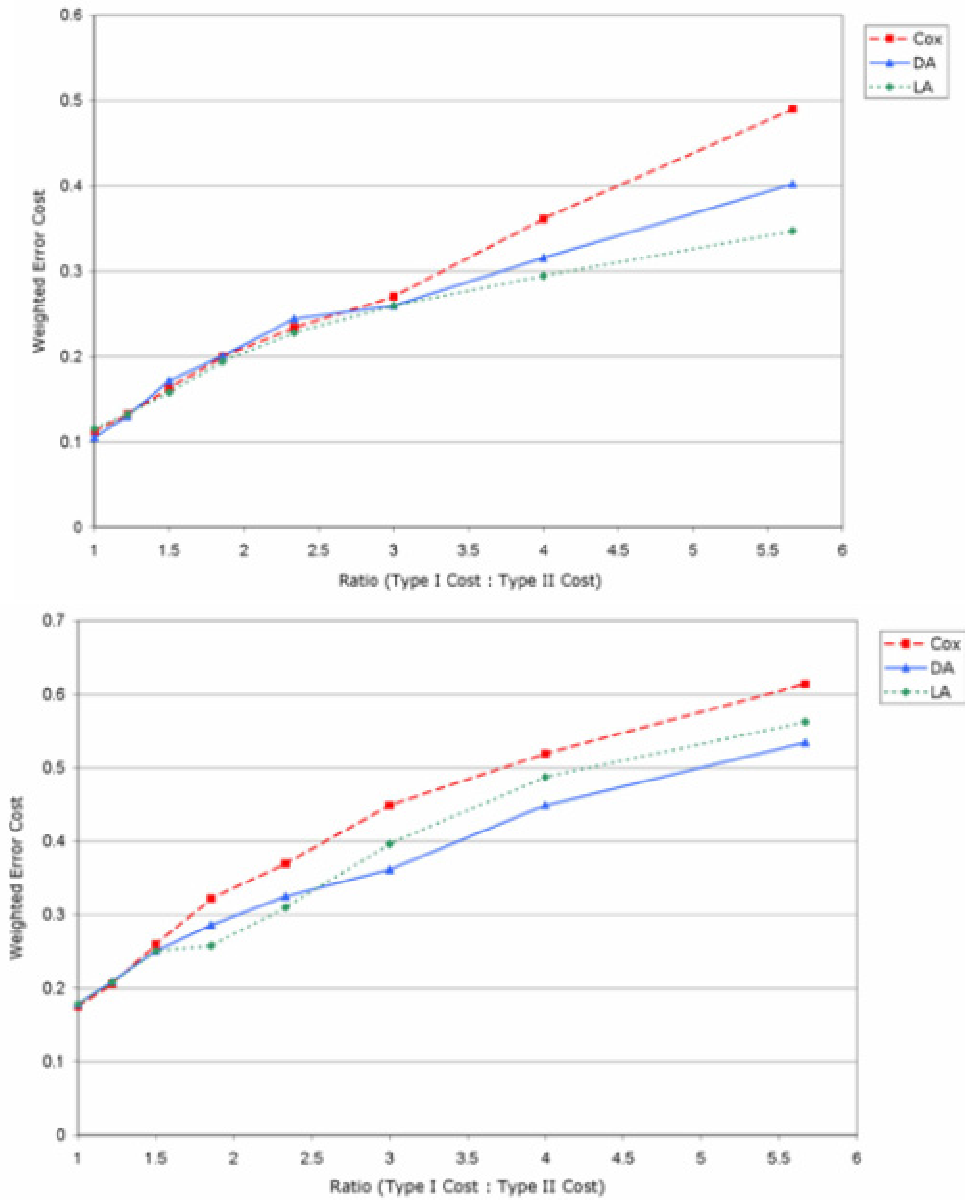
- Two Years (Figure 8): Although the cost of the Cox model increases with the cost of Type I Error compared with the LA model, the Cox model actually became slightly better than the DA model for the highest Type I Error cost.

Figure 8: The cost of two year ahead predictions.



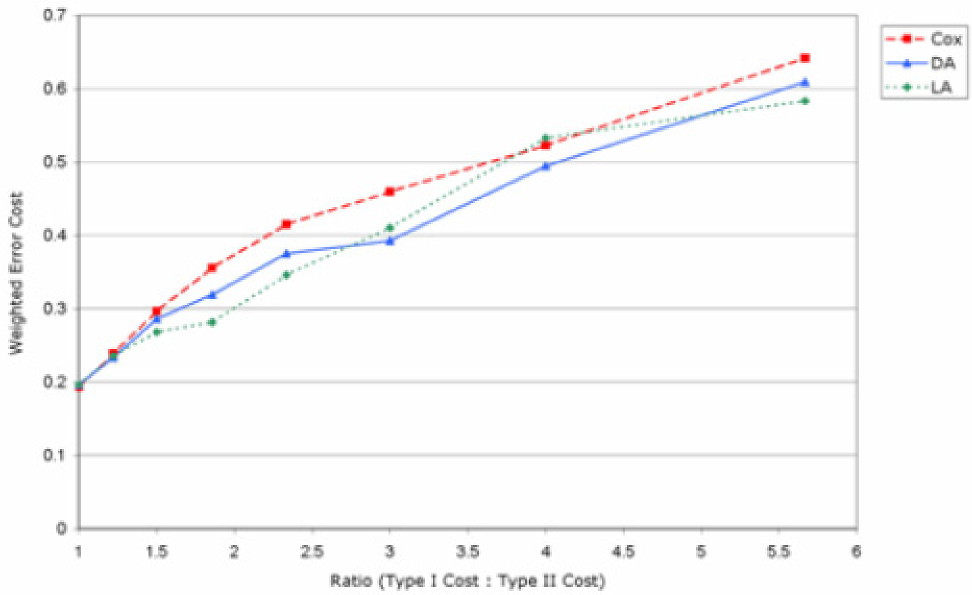
- Three to Eight Years (Figure 9): The Cox model became very costly relative to DA and LA for higher Type I Error Costs, but was comparable to both the other models for lower Type I Error costs. As the prediction interval increased, the Cox model became costlier than the LA and DA models starting from lower Type I Error costs.

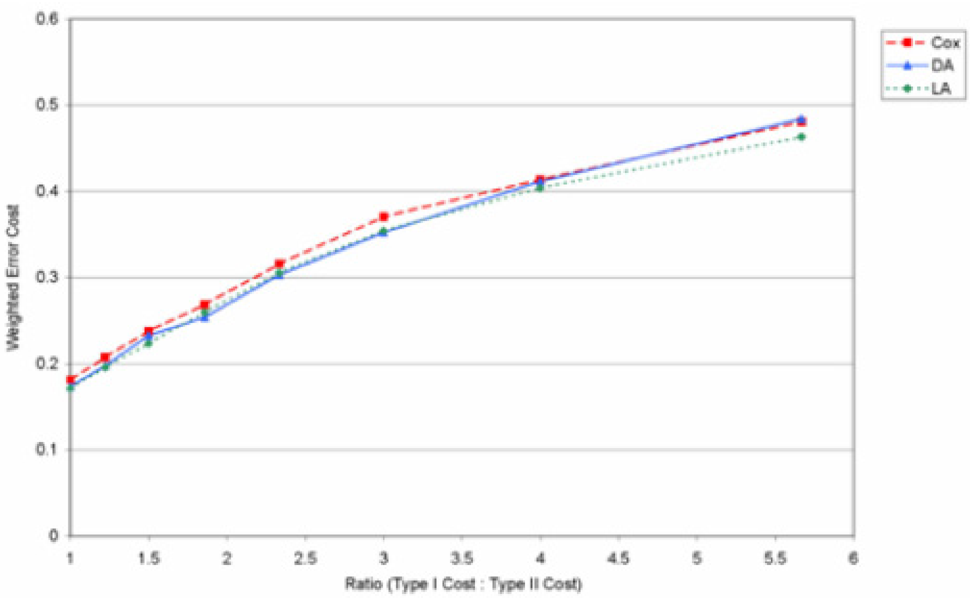
Figure 9: The cost of four (top) and eight (bottom) year ahead predictions.



- Nine and Ten Years (Figure 10): The Cox model followed a similar pattern of becoming the most costly alternative for error cost ratios of above 1.5, but it adapted better to the highest Type I Error costs. The Cox model had better prediction accuracy than LA for the second highest cost of Type I Error for nine year predictions, and DA for the highest cost of Type I Error for ten year predictions.

Figure 10: The cost of nine (top) and ten (bottom) year ahead predictions.





The technique with the best predictive ability varies between DA and LA depending upon the prediction intervals and misclassification costs. It is not possible to choose an overall ‘best technique’ as the prediction interval and misclassification costs are factors specific to each particular situation. Nevertheless, if a ranking must be made the LA would be slightly superior to DA in terms of prediction accuracy as it was superior in more situations and, on average, adapted better to rising Type I Error costs.

Although the Cox model was comparable in many situations, the predictive power of the Cox model was slightly worse than DA and LA, especially for high costs of Type I Error and up to seven year prediction intervals. Furthermore, the relative performance of the Cox model as the prediction level increased was dependent upon the misclassification costs. For lower Type I Error costs there was no improvement, or even a decline, in the prediction accuracy of the Cox model as the prediction interval increased. However with higher Type I Error costs, the prediction accuracy of the Cox model improved for the longer prediction intervals.

Hybrid Models

In most cases, the stepwise procedure for the LA-Cox hybrid model did not even include the survival probabilities as an explanatory variable in the LA model. The survival probability was only included by the stepwise procedure for the 10 year ahead predictions. Furthermore, the hybrid model that was developed for 10 year ahead predictions did not adapt as well to higher costs of Type I Error, as it produced more Type I Error than the simple LA model without a corresponding reduction in Type II Error for both classification and prediction.

The survival probabilities were included in all the DA-Cox hybrid models, but the results were similar to the 10 year LA-Cox hybrid model. For nearly all cases, the weighted error cost measure was higher for the DA-Cox hybrid model than the simple DA model. The reason for this was that the Type I Error had increased in the hybrid model, particularly for higher Type I Error costs and longer prediction intervals.

Overall, this method of producing hybrid models did not improve the ability to classify or predict business failure, and consequently is not appropriate for classifying or predicting business failure.

Analysis of the Importance of Variables

The simplest analysis of variable importance is obtainable from the Cox model, as it was the one model capable of being used for all the different prediction intervals. The details of the variables included in the Cox model are presented in Table 2, where the Exp (Δ) column represents the estimated percentage change in the hazard of failure for a 0.1 increase in a given explanatory variable. Therefore, this value can be used to determine the importance of each variable, where a higher (absolute) value indicates that the variable has a larger impact on the failure of a business. Hence, it is clear that the financial leverage ratio of long-term liabilities to total assets (LTL/TA) is the most important variable, whereby for each 0.1 increase in the ratio the hazard of failure will increase by 42.95%. It is also observable that a range of variable types are shown to be important in the Cox model, as is the case for most BFP models.

Table 2: Variables included in the Cox model, ordered by importance from the top down. Note that all the variables are significant at the 1% significance level.

Variable	Variable Type	Coefficient	Significance	Exp(Δ)
LTL/TA	Financial Leverage	1.667	0.00	42.95%
OPI/TA	Profitability	6.855	0.00	9.99%
RCV/CA	Management Efficiency	1.482	0.00	7.73%
OPI/FA	Profitability	0.466	0.00	5.93%
CA/TA	Liquidity	0.718	0.00	5.12%
Ln(SLS)	Business Size	0.586	0.00	4.43%
RE/TA	Profitability	0.492	0.00	3.88%
QA/CL	Liquidity	0.349	0.00	2.95%
Ln(EMPL)	Business Size	0.202	0.00	2.23%
MVE/TL	Market Structure	0.100	0.00	0.95%

Conclusions

The Cox model was able to classify and predict business failure as well as both DA and LA models for equal misclassification costs. However, the hybrid models were not appropriate for classifying or predicting business failure. This does not mean however that there are not other ways of developing hybrid models that could be appropriate for business failure prediction. DA and LA adapted better to higher Type I Error costs, but the prediction accuracy of the Cox model was comparable with DA and LA when the Type I Error cost was 1.5 times greater than Type II Error cost. The ability to adapt to high Type I Error costs also improved for the Cox model, relative to DA and LA, as the prediction interval increased. It should also be noted that the DA and LA models had very similar classification and predictive ability as has been found in many previous studies.

In addition to the prediction accuracy of the Cox model being comparable with the major techniques (DA and LA), it provides more information about the business failure process through the interpretation of the hazard and survival function over time. In addition, for the situation of different prediction intervals, the Cox model is much easier to interpret and assess the importance of variables as only one model is needed to cater for different prediction intervals.

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Appendix A: In-sample Classifications

Table A.1: The breakdown of the in-sample classifications of each technique into correct classifications (OK), Type I Error (I) and Type II Error (II). This table includes prediction intervals of 1 to 5 years. The number written following each technique's name represents the prediction interval, for example, the heading Cox3 represents a prediction interval of 3 years with the Cox model.

	Cox1			**Cox2**			**Cox3**			**Cox4**			**Cox5**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2552	113	4	2489	169	11	2436	219	14	2378	266	25	2323	312	34
0.55	2550	112	7	2488	165	16	2438	210	21	2374	259	36	2320	301	48
0.6	2549	111	9	2495	157	17	2433	204	32	2368	250	51	2323	276	70
0.65	2548	107	14	2495	150	24	2426	197	46	2360	234	75	2307	254	108
0.7	2545	103	21	2485	145	39	2412	188	69	2350	208	111	2294	223	152
0.75	2546	97	26	2468	140	61	2390	163	116	2316	181	172	2242	186	241
0.8	2535	88	46	2439	122	108	2334	143	192	2246	146	277	2163	146	360
0.85	2500	82	87	2355	105	209	2240	102	327	2100	106	463	1990	110	569
	DA1			**DA2**			**DA3**			**DA4**			**DA5**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2561	77	31	2517	125	27	2458	175	36	2401	229	39	2346	278	45
0.55	2557	76	36	2517	120	32	2461	169	39	2399	222	48	2348	260	61
0.6	2557	75	37	2515	117	37	2458	163	48	2400	209	60	2349	242	78
0.65	2557	71	41	2511	113	45	2451	156	62	2392	198	79	2344	226	99
0.7	2555	70	44	2505	110	54	2432	149	88	2379	182	108	2319	205	145
0.75	2549	67	53	2494	107	68	2417	140	112	2364	168	137	2293	175	201
0.8	2543	65	61	2479	102	88	2394	130	145	2333	147	189	2232	152	285
0.85	2531	63	75	2450	91	128	2361	107	201	2269	119	281	2129	121	419
	LA1			**LA2**			**LA3**			**LA4**			**LA5**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2567	89	13	2515	136	18	2467	184	18	2408	234	27	2359	271	39
0.55	2566	88	15	2514	132	23	2467	178	24	2406	224	39	2359	255	55
0.6	2566	81	22	2518	127	24	2462	171	36	2405	209	55	2345	242	82
0.65	2562	79	28	2519	120	30	2454	160	55	2390	196	83	2343	216	110
0.7	2562	75	32	2511	115	43	2441	148	80	2377	180	112	2308	191	170
0.75	2558	73	38	2495	103	71	2413	136	120	2352	151	166	2267	165	237
0.8	2549	67	53	2470	90	109	2375	118	176	2277	128	264	2181	129	359
0.85	2509	62	98	2420	75	174	2288	89	292	2145	99	425	2032	99	538

Appendix A: In-sample Classifications

Table A.2: This table extends table A.1 to include prediction intervals of 6 to 10 years.

	Cox6			Cox7			Cox8			Cox9			Cox10		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2275	349	45	2248	364	57	2217	382	70	2206	391	72	2185	401	83
0.55	2279	322	68	2239	343	87	2226	348	95	2213	355	101	2194	355	120
0.6	2269	300	100	2245	309	115	2224	313	132	2207	308	154	2189	309	171
0.65	2268	266	135	2242	269	158	2212	265	192	2186	260	223	2175	260	234
0.7	2247	225	197	2208	227	234	2172	225	272	2138	219	312	2113	215	341
0.75	2189	181	299	2147	179	343	2092	181	396	2056	181	432	2030	175	464
0.8	2092	141	436	2022	147	500	1976	136	557	1938	123	608	1907	114	648
0.85	1914	99	656	1845	91	733	1795	81	793	1755	75	839	1722	72	875
	DA6			**DA7**			**DA8**			**DA9**			**DA10**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2316	294	59	2268	339	62	2242	351	76	2208	368	93	2206	355	108
0.55	2314	276	79	2275	310	84	2243	324	102	2217	334	118	2212	317	140
0.6	2319	254	96	2271	280	118	2241	287	141	2209	296	164	2192	287	190
0.65	2299	236	134	2250	254	165	2225	252	192	2186	258	225	2197	240	232
0.7	2280	214	175	2224	217	228	2186	218	265	2166	222	281	2147	215	307
0.75	2232	185	252	2169	187	313	2145	184	340	2106	185	378	2079	175	415
0.8	2174	151	344	2096	148	425	2053	149	467	2013	147	509	1982	137	550
0.85	2063	121	485	1969	118	582	1914	108	647	1875	102	692	1834	98	737
	LA6			**LA7**			**LA8**			**LA9**			**LA10**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	2317	306	46	2266	335	68	2236	350	83	2217	346	106	2211	341	117
0.55	2310	283	76	2263	306	100	2234	310	125	2232	300	137	2214	305	150
0.6	2305	262	102	2268	269	132	2245	268	156	2234	257	178	2205	265	199
0.65	2285	237	147	2240	244	185	2221	233	215	2191	238	240	2176	233	260
0.7	2250	203	216	2215	201	253	2164	205	300	2141	204	324	2120	200	349
0.75	2184	173	312	2145	170	354	2095	168	406	2078	160	431	2051	164	454
0.8	2100	133	436	2037	134	498	2003	129	537	1967	126	576	1951	120	598
0.85	1953	95	621	1889	99	681	1857	91	721	1843	86	740	1811	81	777

Appendix B: Hold-out Predictions

Table B.1: Breakdown of the hold-out predictions of each technique into correct predictions (OK), Type I Error (I) and Type II Error (II). This table includes prediction intervals of 1 to 5 years. The same naming convention as in Appendix A has been used.

	Cox1			Cox2			Cox3			Cox4			Cox5		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	272	13	0	265	20	0	258	27	0	253	32	0	247	35	3
0.55	272	13	0	265	20	0	259	26	0	254	30	1	247	34	4
0.6	272	13	0	265	20	0	260	25	0	253	29	3	248	32	5
0.65	272	13	0	266	19	0	258	23	4	252	28	5	249	30	6
0.7	272	13	0	266	18	1	258	22	5	253	26	6	249	28	8
0.75	273	12	0	263	17	5	258	20	7	254	23	8	245	27	13
0.8	274	11	0	261	16	8	257	18	10	248	22	15	237	24	24
0.85	269	10	6	260	13	12	252	16	17	234	19	32	222	18	45
	DA1			**DA2**			**DA3**			**DA4**			**DA5**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	271	9	5	266	15	4	259	21	5	255	28	2	247	32	6
0.55	269	9	7	264	15	6	257	21	7	254	28	3	249	29	7
0.6	269	9	7	264	15	6	258	20	7	250	28	7	249	28	8
0.65	269	8	8	263	14	8	255	20	10	250	26	9	250	27	8
0.7	269	8	8	263	14	8	253	20	12	250	26	9	249	27	9
0.75	270	7	8	262	14	9	254	18	13	253	21	11	245	25	15
0.8	269	7	9	258	14	13	256	16	13	252	19	14	245	17	23
0.85	269	7	9	258	13	14	254	15	16	245	16	24	243	12	30
	LA1			**LA2**			**LA3**			**LA4**			**LA5**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	274	10	1	267	16	2	260	21	4	252	27	6	247	32	6
0.55	274	10	1	265	15	5	259	20	6	253	26	6	247	31	7
0.6	274	9	2	266	14	5	258	20	7	253	26	6	247	30	8
0.65	273	9	3	264	14	7	258	20	7	251	25	9	247	29	9
0.7	271	9	5	263	14	8	255	20	10	252	24	9	249	27	9
0.75	269	9	7	263	14	8	254	20	11	253	21	11	247	22	16
0.8	271	7	7	260	13	12	255	17	13	249	16	20	243	17	25
0.85	269	7	9	260	12	13	252	10	23	242	12	31	234	13	38

Appendix B: Hold-out Predictions

Table B.2: This table extends table B.1 to include prediction intervals of 6 to 10 years.

	Cox6			Cox7			Cox8			Cox9			Cox10		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	242	39	4	238	43	4	235	46	4	230	50	5	223	56	6
0.55	244	37	4	241	40	4	236	44	5	228	50	7	222	55	8
0.6	245	35	5	242	38	5	233	44	8	225	49	11	221	52	12
0.65	246	33	6	238	38	9	230	43	12	223	46	16	219	47	19
0.7	243	32	10	237	35	13	229	37	19	220	40	25	215	43	27
0.75	235	31	19	230	30	25	223	33	29	218	32	35	215	34	36
0.8	229	24	32	220	25	40	215	26	44	214	26	45	205	27	53
0.85	218	17	50	207	19	59	194	18	73	186	18	81	183	18	84
	DA6			**DA7**			**DA8**			**DA9**			**DA10**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	244	35	6	238	42	5	234	45	6	229	50	6	228	52	5
0.55	244	34	7	241	38	6	235	44	6	229	48	8	223	51	11
0.6	246	32	7	242	36	7	235	43	7	227	47	11	221	50	14
0.65	244	32	9	241	35	9	236	38	11	230	42	13	224	40	21
0.7	245	29	11	241	33	11	235	32	18	226	36	23	224	35	26
0.75	247	25	13	236	29	20	232	25	28	229	28	28	221	32	32
0.8	238	22	25	236	20	29	226	23	36	216	24	45	206	27	52
0.85	232	14	39	222	15	48	212	17	56	200	19	66	191	22	72
	LA6			**LA7**			**LA8**			**LA9**			**LA10**		
Cut-off	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**	**OK**	**I**	**II**
0.5	243	36	6	240	39	6	234	44	7	229	49	7	224	54	7
0.55	243	35	7	240	38	7	235	42	8	228	46	11	221	50	14
0.6	243	34	8	241	37	7	233	39	13	229	41	15	226	44	15
0.65	246	31	8	241	32	12	238	31	16	233	33	19	226	38	21
0.7	243	28	14	244	25	16	234	28	23	229	32	24	221	37	27
0.75	248	22	15	240	23	22	226	27	32	224	28	33	219	31	35
0.8	242	17	26	227	22	36	215	23	47	208	25	52	205	26	54
0.85	226	11	48	217	15	53	204	17	64	198	17	70	192	17	76