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ABSTRACT

Analysis of the difference between the lowest and second lowest bids, or bid-spread, in a 'lowest wins' auction is of possible value in strategic bidding; providing an indication of mistakes in bids; determining a justifiable amount of bid security; and a means of providing some insight into the consequences of non-traditional auction arrangements. Bid-spread analysis, as developed in this paper, provides some explanations concerning the nature of bids and their statistical properties. In particular, it is shown here that, through the analysis of several datasets originating in various parts of the world, the percentage bid-spread is consistent with the assumption that bids are entirely random, being drawn from a lognormal distribution. The high values of the correlation coefficients, together with the failure of the two most popular correlates - contract size value and number of bidders – to account for any significant trends once the order statistic effects are removed provides overwhelming evidence in favor of the dominance of inherent variability in bidding.

Keywords: Bidding, tendering, bid-spread, spread, money left on the table, Vickrey Auctions, statistical models.

INTRODUCTION

Bid-spread, also variously termed money "left on the table" (Gates, 1960: 13), "The Spread" (Park and Chapin, 1992: 187) or just plain "spread" (Gates, 1960: 13), is concerned with the difference between the lowest and second lowest bids. For bidders, the primary interest in bid-spread is that it is "… the amount by which the lowest tenderer is underbidding the second lowest tenderer and which therefore is foregone profit" (Runeson, 1987: 103). This has led to its development as a possible aid in strategic lump sum or unit price bidding (Gates, 1960) although the former has been criticized as resulting in fewer jobs won, with uncertain consequences (Park and Chapin, 1992: 188). Bid-spread has also the potential for use in identifying mistakes in bids and determining the maximum justifiable amount of bid security (Gates, 1960).

From a theoretical perspective, the "remarkable" sensitivity of strategic bid-spread-based bidding to the *status quo* of contract auctions won and lost has led Runeson (1987) to the conclusion that bidders are very much market oriented in their pricing, making the 'foregone profit' unrecoverable in the face of the economic pressures involved. This leads to the conclusion that traditional lowest wins sealed bid auctions must produce subnormal profits in the long run due to the difference between the lowest and second lowest bids – a conclusion also reached by Nobel Laureate William Vickrey in his general analysis of highest wins[1] auctions (Vickrey, 1961). Despite these, what are to date, uncontested results there have been no apparent changes in the still predominant competitive bidding mode of construction procurement. Vickrey's suggested correction to overcome the problem – that of awarding the contract to the lowest bidder at the second lowest price (now popularly termed the Vickrey

[1] The symmetry of highest and lowest wins auctions makes the distinction analytically trivial

Auction) – has never even been trialed. Why this is the case is not clear, though probably due to the counterintuitive nature of the objections to the lowest bid criterion.

In this paper, we reexamine the empirical nature of bid-spread, its modeling, explanation and prediction through the analysis of seven datasets, Runeson's interpretation and the Vickrey solution.

DATA

Seven separate datasets were analyzed. Each dataset, except dataset 7 for which only the lowest and second lowest bid values were available, comprised the values of all the bids for each contract auction, updated to the first quarter 1980 sterling equivalent by the relevant price indexes and exchange rate series'. Details of the datasets and summary statistics of the updated bids are given in Table 1.

Data set	Source	Type	Period	No of auctions	Average low bid	Average Std Devn	Average Coeff of Varn
1	Skitmore (1986)	London building contracts	1981-2	51	1.57m	82k	5.52
2	Skitmore (1986)	London building contracts	1976-7	373	0.81m	47k	6.36
3	Brown (1986)	USA Govt agency building contracts	1976-84	62	1.39m	119k	19.06
4	Runeson (1987)	Australian PWD contracts	1972-82	152	1.51m	103k	6.98
5	Runeson (1987)	Australian PWD specialist contracts	1972-82	161	0.21m	29k	16.21
6	Skitmore (1981)	UK building contracts	1969-78	272	0.81m	48k	6.03
7	Drew (1995)	Hong Kong Govt building contracts	1981-90	199	0.96m	-	-
All				1270	0.90m	59k	8.54

Table 1: Datasets

EMPIRICAL ANALYSIS (1)

Gates' (1960) analysis of bid results published by the USA State of Connecticut for 1957 to 1959 and the states of New Hampshire and Vermont from 1958 to 1959 found, "by the method of least squares [and log-log paper], the best fitting curve" (p18), to be

$$p = 108C^{-0.266} \qquad \textbf{\textit{(1)}}$$

where p and C denote the percentage bid-spread and lowest bid value respectively. In terms of dollars this is

$$B = 1.08C^{0.734} \qquad \textbf{\textit{(2)}}$$

where B denotes the average (dollar) bid-spread.

In contrast, Runeson (1987) compared the mean bid/cost estimate ratios for the lowest and second lowest bid against the number of bids in the auction for his own dataset of 265 contract auctions, showing that not only do the average ratios for the lowest and second lowest bids reduce as the numbers of bidders in the auction increase, but that the average difference between the ratios also reduces with increasing numbers of bidders. A simpler version of this finding was reported by Park and Chapin (1992) in their analysis of "60 recent jobs" in which they found the average bid-spread to be 8.0, 5.8, 3.8 and 2.0 percent on jobs having 4 to 6, 7 to 9, 10 to 12 and 13 to 15 bidders respectively, suggesting a linear model to be appropriate.

It is clear, therefore, from a purely empirical perspective (none of the previous research having proposed *a priori* reasons) that two likely explanatory variables exist – the contract size value (as measured by Gates' C) and the number of bids in the contract auctions, to which we will refer for convenience sake as n[2]. Interestingly, none of the previous studies considered both variables, with Gates concentrating on C to the exclusion of n, and Runeson and Park and Chapin concentrating on n to the exclusion of C.

Gates has also shown that transformations of both the dependent variable (bid-spread) and the independent variables are likely to be beneficial, having used the raw (dollar) bid-spread, percentage spread, log transformed spread, raw C and log C.

To subject the data to as rigorous analysis as possible and yet avoid logistical and computational overload we concentrated on three forms of dependent variable (raw, log and percent bid-spread) and four forms of each of the two independent variables (raw, square root, reciprocal and log – in order of increasing strength of transformation).

In addition, in view of the lack of any theoretical basis for the choice of independent variables, one extra variable was added based on the properties of the order statistics uniform probability density function (rectangular distribution). Letting s be the standard deviation of the uniform pdf, then the range, $r = s\sqrt{12}$. The difference between the expected value of the lowest and the expected value of the second lowest of a sample size n drawn from this distribution is $\gamma = r(n+1)^{-1}$, which is equivalent to $\gamma = s\sqrt{12}(n+1)^{-1}$. Of course, the γ here is based on expected values and is therefore only a mean value, i.e., even if the data were drawn from a uniform distribution, γ would be the average result over a long set of trials and the results of each trial

[2] Thus for an *n-size* auction, there are n number of bids entered.

would bound to differ from this central tendency. However, γ was included nevertheless as an indicator of the likely amount of randomness of the bid-spread.

The full set of Pearson correlation coefficients for all the datasets and indication of their significance (pr<0.05) are given in Tables 2a-c. For Table 2a, which is concerned with the raw bid-spread, the value of the lowest bid, contract size value provides the most consistently high correlation, with the raw C and all three transformations being significant in all but one case (C^{-1} for dataset 4). Of these, C, $\sqrt{C}$, $\mathbf{ln}C$ and C^{-1} recorded the highest coefficients 4, 2, 1 and 0 times respectively, the order also reflected in the ranked correlation coefficients for the pooled (All) data. The number of bidders is not significantly correlated in any except dataset 7, where the raw n and its three transformations are all significant, although in most cases less so that contract size value. The expected value variable, γ, is however, highly correlated, and records the highest coefficient of all the independent variables for four out of the six datasets on which it was used, with the extraordinary value of 0.9694 for dataset 6. It is also interesting to note that the highest three γ correlations were for the three UK datasets. Again, the results of the pooled data analysis gives a good reflection of the order of importance of the independent variables, with the top ranked γ followed by C, $\sqrt{C}$ and $\mathbf{ln}C$ respectively in order of strength of correlation.

Dataset	n	$\sqrt{n}$	n^{-1}	$\mathbf{ln}n$	C	$\sqrt{C}$	C^{-1}	$\mathbf{ln}C$	γ
1	-0.0453	-0.0614	0.1030	-0.0769	*0.6175*	*0.5917*	*-0.3627*	*0.5247*	*0.8168*
2	0.0434	0.0521	-0.0736	0.0605	*0.4147*	*0.4544*	*-0.1770*	*0.3956*	*0.7333*
3	-0.1080	-0.1124	0.1339	-0.1188	*0.7386*	*0.7817*	*-0.3345*	*0.7127*	*0.6994*
4	0.0384	0.0453	-0.0380	0.0475	*0.7588*	*0.6767*	-0.1727	*0.5238*	*0.6942*
5	-0.1152	-0.1127	0.0970	-0.1085	*0.2811*	*0.4040*	*-0.2453*	*0.4327*	*0.6039*
6	-0.0480	-0.0431	0.0176	-0.0361	*0.9007*	*0.7277*	*-0.1339*	*0.4496*	*0.9706*
7	*-0.2517*	*-0.2909*	*0.4382*	*-0.3361*	*0.6032*	*0.5679*	*-0.2679*	*0.4846*	*na*
All	-0.0036	-0.0011	-0.0030	0.0009	*0.7263*	*0.6164*	*-0.0868*	*0.4300*	*0.8416*

Italics = significant at 5% level

Table 2a: Correlations: raw bid-spread

Table 2b gives the results for the **ln** bid-spread dependent variable. These are quite similar to the results for raw bid-spread, with all the contract size values significant and higher than the number of bidders variables. The log transformed γ is again the best correlate for four of the six datasets analyzed and again provides the highest correlation for the pooled data. Again dataset 3 is different, but with $\sqrt{C}$ and **ln**C the best this time. **Ln**C also provides the best correlation for dataset 4.

Dataset	n	$\sqrt{n}$	n^{-1}	**ln**n	C	$\sqrt{C}$	C^{-1}	**ln**C	**ln**γ
1	0.0684	0.0576	-0.0164	0.0449	*0.4753*	*0.5344*	*-0.5363*	*0.5622*	*0.5731*
2	0.1004	*0.1030*	-0.1009	*0.1040*	*0.3521*	*0.4561*	*-0.2756*	*0.4688*	*0.6537*
3	-0.2307	-0.2454	*0.2641*	*-0.2566*	*0.5988*	*0.7080*	*-0.4770*	*0.7555*	*0.6965*
4	0.0375	0.0527	-0.0577	0.0618	*0.5307*	*0.6198*	*-0.3928*	*0.6347*	*0.6321*
5	*-0.2171*	*-0.2187*	*0.2057*	*-0.2173*	*0.2742*	*0.4212*	*-0.4291*	*0.5338*	*0.6611*
6	0.0293	0.0274	-0.0263	0.0262	*0.4702*	*0.5789*	*-0.3490*	*0.5890*	*0.6666*
7	*-0.2034*	*-0.2274*	*0.2824*	*-0.2501*	*0.4841*	*0.5172*	*-0.3413*	*0.4975*	*na*
All	0.0133	0.0166	-0.0154	0.0181	*0.4433*	*0.5556*	*-0.2712*	*0.5752*	*0.6737*

Italics = significant at 5% level

Table 2b: Correlations: ***ln*** *bid-spread*

Table 2c gives the results for the percentage bid-spread (percentage second lowest bid is above the lowest bid) dependent variable. Here, the results are more variable, with dataset 1 having no significant correlations and datasets 3 and 4 having only one significant correlation (that of γ%). Across all the individual datasets, γ% provides the highest correlation for all the 6 datasets for which it was computable, as well as recording the highest correlation for the pooled data.

The big surprise here is that the contract size value variable, or any of its transformations, is significantly correlated at all. In fact, in all cases, the average percentage bid-spread **decreases** with increasing contract size value, suggesting that percentage bid-spread is not exactly proportional to contract size value, as is implied by ***(1)***. One possible cause of this might be that the C results are being confounded with the n. In other words, larger contract size values attract larger number of bidders. This possibility can be tested in several ways. The simplest is to use a trivariate regression analysis that includes the suspected confounding variables.

Dataset	n	$\sqrt{n}$	n^{-1}	**ln***n*	C	$\sqrt{C}$	C^{-1}	**ln***C*	γ%
1	-0.1235	-0.1213	0.0001	-0.1181	-0.0448	-0.0789	0.0959	-0.1064	0.2621
2	*-0.3106*	*-0.3126*	*0.3462*	*-0.3382*	*-0.1416*	*-0.2214*	*0.3602*	*-0.3078*	*0.8175*
3	-0.0953	-0.1191	0.1751	-0.1413	-0.2033	-0.2268	0.2438	-0.2474	*0.3106*
4	-0.1460	-0.1448	0.1245	-0.1406	-0.0087	-0.0409	0.0631	-0.0758	*0.3616*
5	-0.1192	-0.1381	*0.1630*	-0.1529	*-0.2212*	*-0.2872*	*0.2076*	*-0.2968*	*0.3518*
6	*-0.1861*	*-0.1927*	*0.1964*	*-0.1968*	-0.0501	*-0.1331*	*0.2561*	*-0.2178*	*0.5750*
7	*-0.3066*	*-0.3352*	*0.3649*	*-0.3566*	*-0.1539*	*-0.2148*	*0.2961*	*-0.2756*	na
All	*-0.1574*	*-0.1860*	*0.2382*	*-0.2115*	*-0.1311*	*-0.2370*	*0.2495*	*-0.3449*	*0.4107*

Italics = significant at 5% level

Table 2c: Correlations: % bid-spread

This was done for all the cases where a form of C was significant in Table 2c and with n^{-1}, the generally highest correlated form of n, and γ%. The results of all the regressions are summarized in Table 3, indicating the coefficient of regression, multiple r (which is directly comparable with the correlation coefficients in Table 2) and the t-significance or otherwise of each of the two independent variables. These show that, for dataset 2, the highest achievable r of around 0.82 has no significant contract size value effect. For dataset 5 highest r values of around 0.39 to 0.41 include several forms of contract size value, while dataset 6, with a best r values of around 0.56 have no significant contract size value form.

Dataset 7, with no standard deviation measure available includes forms of both contract size value and number of bidders, with the all best models for the total pooled datasets, with an r value of around 0.43 to 0.48 including contract size value forms.

It is not easy to compare between the three sets of results in Tables 2a-c as each uses a different dependent variable. Which model is best depends on the purpose of the model and, ultimately,

Dataset	Indep var 1	Sig	Indep var 2	Sig	r
2	C	No	n^{-1}	Yes	0.3539
	C	No	γ%	Yes	0.8211
	$\sqrt{C}$	Yes	n^{-1}	Yes	0.3620
	$\sqrt{C}$	No	γ%	Yes	0.8213
	C^{-1}	Yes	n^{-1}	Yes	0.4073
	C^{-1}	No	γ%	Yes	0.8216
	LnC	Yes	n^{-1}	Yes	0.3798
	LnC	No	γ%	Yes	0.8215
5	C	Yes	n^{-1}	No	0.2623
	C	Yes	γ%	Yes	0.3915
	$\sqrt{C}$	Yes	n^{-1}	No	0.3220
	$\sqrt{C}$	Yes	γ%	Yes	0.4110
	C^{-1}	Yes	n^{-1}	Yes	0.2804
	C^{-1}	No	γ%	Yes	0.3512
	ln$C$	Yes	n^{-1}	Yes	0.3440
	ln$C$	Yes	γ%	Yes	0.3976
6	$\sqrt{C}$	No	n^{-1}	Yes	0.2224
	$\sqrt{C}$	No	γ%	Yes	0.5648
	C^{-1}	Yes	n^{-1}	No	0.2788
	C^{-1}	No	γ%	Yes	0.5639
	ln$C$	Yes	n^{-1}	Yes	0.2590
	ln$C$	No	γ%	Yes	0.5656
7	C	Yes	n^{-1}	Yes	0.4186
	$\sqrt{C}$	Yes	n^{-1}	Yes	0.4430
	C^{-1}	Yes	n^{-1}	Yes	0.4591
	ln$C$	Yes	n^{-1}	Yes	0.4650
All	C	Yes	n^{-1}	Yes	0.2516
	C	Yes	γ%	Yes	0.4345
	$\sqrt{C}$	Yes	n^{-1}	Yes	0.3031
	$\sqrt{C}$	Yes	γ%	Yes	0.4549
	C^{-1}	Yes	n^{-1}	Yes	0.3513
	C^{-1}	Yes	γ%	Yes	0.4298
	ln$C$	Yes	n^{-1}	Yes	0.3913
	ln$C$	Yes	γ%	Yes	0.4868

Table 3: Trivariate analyses of percentage bid-spread assuming uniform distributions

the loss function involved. What does seem to emerge from this analysis, however, is the importance of the random indicator, γ, as it figures prominently in every case. The statistic, γ, it will be recalled, was devised as a simple measure of randomness based on the assumption of a uniform distribution of bids and, as an expected value, was not expected to account for all the variance in the data. In the next part of the analysis, we examine this variable in more detail against some obvious alternatives to offer some causal explanation of the statistical nature of bid-spread.

THE STATISTICAL HYPOTHESIS

The large literature on bidding theory and models (see Stark and Rothkopf, 1979, for an early bibliography) is replete with what can be termed 'the statistical hypothesis' in that auction bids are assumed to contain statistical properties such as fixed parameters and randomness. The first contributions (e.g., Friedman, 1956) assumed that each bidder drew bids from a probability distribution unique to that bidder, with low frequency bidders being pooled as a special case. Later work by McCaffer and Pettitt (1976) and (Mitchell, 1977) for example, assumed the probability distributions to be non-unique and homogeneous, enabling a suitable distribution shape to be empirically fitted (uniform, in the case of McCaffer and Pettitt) and the derivation of order statistics based on an assumed (normal) density function. Later empirical work by Skitmore (1991) showed the homogeneity assumption to be untenable for his three datasets of construction contract auctions, at least insofar as its superiority in predicting the probability of lowest bidders is concerned (Skitmore, 1999). Runeson and Skitmore (1999), however, have cast doubt on the whole future of the heterogeneous approach to modeling construction contract auction bids on the basis of its necessary, but forced, assumption of temporal invariance (fixed parameters) in the absence of the lengthy repeated trials assumed by the statistical model – each bidder not bidding frequently enough to generate a reasonable size dataset.

The assumption of homogeneity, of course, solves the problem in a stroke as, if each bidder is assumed to bid from **the same distribution**, all bids made by all bidders contribute to the empirical estimation of the parameters, the increased amount of data made available this way thus enabling the temporal invariance assumption to be relaxed at least to a yearly time span. Therefore, if we were to be able to determine the type of distribution, and its parameters, perhaps on a yearly basis, it would then be possible to predict the order statistics involved and hence the value of the bid-spread. That the correlation results for the statistic γ have been so successful so far in this analysis suggests this may be feasible. However, there are a few difficulties in the way before this can be exploited. One of these is to specify the type of probability distribution involved. Another is to predict its parameters.

Several types of probability distributions have been considered to be appropriate for construction contract auction bids. Of these, only two have been fully tested empirically. In the first of these, as has already been mentioned, McCaffer and Pettitt (1976) found the uniform distribution function to be the best fit for a set of Belgian construction contract auction data. In the second, Skitmore (1986)'s replication of this with three sets of UK data indicated a three parameter[3] lognormal to be the most appropriate, with a non-constant second moment (variance) that is possibly a function of the first moment (average bid) for each contract auction. Being positively skewed, the lognormal assumption satisfies the corresponding qualitative findings of at least two other empirical studies of construction contract auction bids (Park, 1966; Beeston, 1974) and intuitions of the majority of theoreticians in the sealed bid field generally (e.g., Arps, 1965; Brown, 1966; Crawford, 1970; Capen *et al*, 1971; Klein, 1976; Weverbergh, 1982). Parameter estimation of lognormal distributions is, of course, a simple matter as all that is needed is to take the log values of the bids and work with the normal distribution thereafter.

[3] Later modified by Skitmore and Pemberton (1994) to the simpler two-parameter lognormal distribution for application in strategic bidding.

EMPIRICAL ANALYSIS (2)

The analysis described above was repeated with γ being replaced by the difference (denoted by the symbol λ) between the expected value of the lowest and expected value of the second lowest order statistics for the lognormal distribution for each auction. Table 4 gives the Pearson correlation coefficients. All are significant at the 5% level

Dataset	Raw spread	Ln spread	% spread
1	*0.7933*	*0.5827*	*0.2886*
2	*0.7312*	*0.6559*	*0.8358*
3	*0.8385*	*0.7630*	*0.4573*
4	*0.7621*	*0.6647*	*0.4339*
5	*0.55540*	*0.6696*	*0.4827*
6	*0.9708*	*0.6693*	*0.6179*
7	-	-	-
All	*0.8640*	*0.6836*	*0.6018*

Table 4: λ correlations

As expected, the correlation coefficients are generally higher for the λ values than were the γ values in Table 2a-c, and particularly so for the percentage bid-spread.

Trivariate analyses were run on each dataset for the percentage bid-spread against all the various forms of the independent variables to investigate the effect of partialling out the effects of the λ values. In all cases, except the small sample dataset 1 with C^{-1} and log C, the λ variable was significantly correlated with the percentage bid-spread. In only two instance, that of $\sqrt{C}$ and C^{-1} for dataset 5 (r=0.5041 and 0.5031 respectively), was a significant result obtained for a non-λ variable. For the pooled (All) dataset, however, all the independent variables except raw C were significant, with r-values ranging from 0.6062 to 0.6124.

CONCLUSIONS

The difference between the lowest and second lowest bids in a ‘lowest wins’ auction is of interest as it represents the lowest bidder’s “foregone profit”, and thus of possible value in strategic bidding; providing an indication of mistakes in bids; determining a justifiable amount of bid security; and a means of providing some insight into the consequences of non-traditional auction arrangements. Bid-spread analysis, as developed in this paper, provides some explanations concerning the nature of bids and their statistical properties. In particular, it is shown here that, through the analysis of several datasets originating in various parts of the world, the percentage bid-spread is consistent with the assumption that bids are entirely random, being drawn from a lognormal distribution. The high values of the correlation coefficients, together with the failure of the two most popular correlates - contract size value and number of bidders – to account for any significant trends once the order statistic effects are removed provides overwhelming evidence in favor of the dominance of inherent variability in bidding.

It should be noted though that, although the effects are qualitatively consistent for all the datasets analyzed, this is not the situation when the datasets are pooled. Why this should be the case is not clear and in need of further study if some universal model is to be found. Meanwhile, the methods used in this paper should be of value in analyzing individual datasets.

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