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BIDDING MODELS: TESTING THE STATIONARITY ASSUMPTION

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ABSTRACT

With notably few exceptions, bidding models contain probability distributions with parameters that are assumed to be fixed, or stationary, over time. Some methods of method of testing the tenability of this assumption are examined and applied to eight datasets. Of particular interest is the statistical significance of two types of periodicity; (1) that bidders gradually reduce their bids prior to winning a contract and (2) that bidders have periods in which they are more competitive and periods in which they are less competitive. To test (1), McCaffer and Pettitt's (1976) cusum method is used and shown to have a limited interpretation in this context. McCaffer's 'deficit' statistic is then used in conjunction with a one-way analysis of variance (ANOVA) and shows (1) to be untenable for the samples involved. To test (2), the deficit statistic is again used with an ANOVA to examine all possible sub series of bids.

Keywords: bidding, behaviour, parameters, cusum method, deficit statistic.

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BACKGROUND

The data demands of bidding models are such that a trade-off is needed between the flexibility of the models and the accuracy of the estimates of the parameters of the models. Apart from a few notable exceptions (Beeston, 1983; Morin & Clough 1969) this has resulted in models being built on the assumption that bidder's behave in a consistent, if probabilistic, way over a reasonably long period of time regardless of changing conditions (Runeson and Skitmore 1999). In particular, stationarity is assumed, that is that the probability distributions used to model the bids have parameters whose values are fixed over this time. The tenability of this assumption has been questioned on theoretical grounds – standard economic theory predicts that, for example, changing workloads and market conditions must have a destabilising influence over time (Flanagan & Norman, 1984; Harris & McCaffer 1983:219; Runeson and Skitmore 1999) – and it is likely that new models will be needed if real-world bidding offers a significant departure

Whether or not the stationarity assumption is reasonable in practice is likely to depend on several factors. One is the extensive use of subcontracting in the industry, which protects main contractors to some extent from workload problems. Another is that the conditions of uncertainty involved in estimating both price and cost levels may make significant systematic adjustments in competitive behaviour difficult, if not impossible. Of the little empirical research to date aimed at establishing the extent to which stationarity exists ***in practice***, Skitmore (1981,1987) and Rawlinson and Raftery (1997) have identified some significant yearly changes in the moments of the overall aggregated distribution of bids – suggesting these to be associated with market conditions. At the level of the individual firm, views seem to be mixed. Shash (1993), for example, found "the need for work" to be one of the biggest factors influencing the bid/no bid decision. Ahmad and Minkarah (1988), on the other hand, give no mention of this. Meanwhile, Griffis' (1992:153) personal experience is that the "volume on hand (backlog, work on hand) is a major influence on the utility that a building contractor places on a particular bid letting" although he does admit this to "have not been proven by the writer … [and providing] fruit for further consideration and research" (Griffis 1992:164).

In fact, one study of individual bidders was reported as early as 1976. This is by McCaffer and Pettitt (1976), in which they examined a set of 185 contracts for building work and 350 contracts for roadwork. This involved the use of the statistics, by large clients, to identify 'outliers' in a set of bids thereby rejecting low bids that could be demonstrated to be unbelievable low. To do this, they looked at the competitiveness of individual contractors through each's mean bid levels, during the course of which:

> It was noticed that some companies, for a year or so, would be consistently bidding low and then would be bidding high for the remainder of the time, thus exhibiting two types of behaviour, which in the long run looked average. A method which should recognise this behaviour is the calculation of cumulative sums (cusums) of ((bid/mean bid)-1). If a company is consistently bidding below the mean bid, then ((bid/mean bid)-1) will be negative and the cusum will drift away from the zero line in a negative direction. If the company then starts to bid above the mean bid, then the cusum will change direction and drift in a positive direction. Thus, one can see at a glance whether a company is behaving competitively (cusum drifting in a

> negative direction) or not (cusum drifting in a positive direction). We expect winning bids to come after a downward drift, which will then be followed by an upward drift, as the company is not so eager to gain a contract … This technique is useful in identifying the companies which are eager to win contracts by offering lower than average bids at a given time. In Table 2, we give the number of times a winning bid was preceded by a fall of a given number of increments in the contractor's cusum value. (p.6)

Percentage of the total number of winning bids preceded by a fall of 'M' increments in the contractors' cusum value (%)	Number of increments (i.e., previous bids) ('M')
85	2
75	3
69	4
65	5

McCaffer and Pettitt's Table 2

In a further publication (Harris and McCaffer 1983:236) concerning the same data is added "An analysis of this type for some 600 contracts involving almost 400 contractors showed that there were only 15% of cases when the winning bidder had a rising graph preceding his winning bid [while] conversely there were 85% of cases when the winning bidder had a declining graph", with the original Table 2 above being presented again as a summary of the results. Although no statistics are provided concerning the significance or otherwise of this result, it is clearly impressive[1].

[1] Since then, similar results have also been obtained in Yiin's (1987) study of subcontractors.

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In yet another paper (McCaffer 1976), McCaffer uses what he terms a 'deficit' statistic, that is the percentage difference between the contractor's bid and the lowest opposing bid. This was subjected to a variety of tests for randomness of his Belgian data leading to the eventual conclusion "that contractors bidding behaviour can be regarded as random" (p3)

Here, we describe the analysis of eight disparate datasets to test the general tenability of the stationarity assumption by answering the question "Do contractors' bids commonly change systematically over a period of time or are they little different from random?" In particular, we develop methods of checking the statistical significance of the periodicity involved. To do this we first start with McCaffer and Pettitt's cusum method and show it to have a limited interpretation in terms of a measure of progress towards producing the winning bid. We then use McCaffer's 'deficit' statistic in a more direct method which is designed to demonstrate the phenomenon more clearly and show that winning bids are not in general preceded by increasingly more competitive bids. This method of analysis is then extended to the various sub series involved and, for the data used, it is shown that bids do seem to group together on occasions. However, rather than confirming McCaffer and Pettitt's observation that there may be two types of bidding behaviour involved, they appear to be more due to the presence of a very few outliers than a continuing trend.

CUSUM ANALYSIS

Datasets

The data needed for the analysis are difficult to obtain for, as Griffis (1992) points out, there is nothing to gain and everything to lose for a bidder to allow his bidding trends to be analysed by competitors. Nevertheless, six datasets were obtained from existing data sources for the cusum analysis. These are summarised in Table1. The first three sets are from Skitmore (1986) and comprise (1) a full set of bids made by a single medium-large size London construction company over a 12 month period in the early 1980s, (2) a full set of bids for all the contracts of a North of England local council over a three-year period again in the early 1980s and (3) a full set of bids for all the contracts in the Great London area collected over a three month period commencing late 1976. The fourth dataset is for a full set of bids for all the building contracts let by a USA Government Aerospace Agency from 1976 to 1984, while the fifth dataset is for a single USA contractor in the late 1960s and published in Shaffer and Micheau (1971). The data for the Shaffer and Micheau's bidder did not constitute all of the work for which he competed during the period but they were for all the projects for which the contractor collected such data. 40% of the bids were made in one year – a year in which the contractor initiated an in-depth study of his bidding practices. The sixth dataset comprised more recent data obtained from the Hong Kong Architectural Services Department for their building contract bids from 1990 to 1996. Apart from the Shaffer and Micheau data, nothing is known of any changing conditions that may have occurred during the periods to which the data relate.

The bids analysed comprised single most recorded bidder from each dataset. Therefore, although it is not known how many other contracts were bid during the period by the contractor concerned, it is likely that nearly all, if not all, his bids for the period are being examined. The original contract sequence numbers are retained as an indication of the frequency of the contractors' bidding record for the particular client/owner involved. In the analysis that follows, the term "winning bid" is taken to mean the lowest bid in the auction.

Results

The results of the cusum analysis are shown in Fig 1. The circles represent the cusum values for the bidder concerned, with the closed circles representing winning bids. Fig1a shows the results for the Skitmore 1 bidder, who generally bids slightly above the mean bid. The Skitmore 2 bidder (Fig 1b), on the other hand, having bid competitively for the first 60 contracts in the database, then bids generally above the mean bid for the next 60 contracts, only to resume bidding competitively thereafter – seemingly offering clear evidence of McCaffer and Pettitt's assertion that some bidder's trends do change over time. The Skitmore 3 bidder also seems to change trend over time, with a trail of more competitive bids during the earlier phase of the period but fluctuating quite wildly in the second half of the period. The USA Govt bidder (Fig 1d) likewise appears to have a different trend in the second half of the series. Closer inspection, however, suggests that all that is really different is two bids in the middle of the series. These move the graph line substantially upwards from whence the trend continues virtually as before. Shaffer and Micheau's bidder (Fig 1e) has a similar disjointed trend, perhaps coinciding with the change in bidding policy noted above. However, it is again the influence of 3 or 4 bids in the middle of the series that gives this impression of disjointedness rather than any real change in overall trend. Finally, the Hong Kong bidder appears to exhibit the most consistent trend of all, with a gently down sloping graph indicating consistent bidding below the mean bid for each contract.

The top half of Table 2 summarises the results of the cusum analysis. This shows the number of times the cusum value of previous bids fell prior to a winning bid. For example, of the six occasions the Skitmore 1 bidder had won a contract, his cusum value had fallen four times on the previous bid, one time on the previous two bids and one time on the previous 5 bids (the figures in brackets show the cumulative results). The equivalent percentage figures are also shown for comparison with McCaffer and Pettitt's results. The Hong Kong results are not shown as nearly all the bidder's cusum values fell throughout the whole series.

Comment

Although at first sight, the cusum values do seem to highlight trends and changes in trends in bidding behaviour, it is clear from the above commentary that they need careful interpretation. For at least two of the bidders, what appears to be a dramatic change in trend is just the effect of a tiny few bids rather than a change in trend. Also, as the cusums only reflect bids below the mean bid, a contractor (such as the Hong Kong contractor) entering bids consistently below the mean bid[2] is virtually impossible to analyse in terms of counting the number of times the cusum value drops prior to a

[2] Such asymmetric behaviour is not unexpected for construction contracts (Bajari 1997; Bajari & Ye 2000) as some firms are better managed than others for "a variety of other reasons" (Bajari 2001:2),

winning bid. McCaffer's 'deficit' statistic, on the other hand, would seem to offer less chance of a result that is an artefact of the statistic as it measures the departure from the main bid of interest, the lowest bid, instead of the mean bid. This is examined in the next section.

DEFICIT ANALYSIS

A simpler and more informative approach is to use McCaffer's 'deficit' value, which is just the percentage the bid is above the lowest bid divided by 100. By comparing the bidder's mean deficit values for the ultimate bids immediately prior to the winning bid with the bidder's mean deficit values for penultimate bids, etc., it is possible to assess the degree of downward drift for individual contractors. Fig 2 provides the results of the 'deficit' analysis. The bidding patterns now become clearer, with the Skitmore 1 bidder being less than 0.3 on all except two occasions. Using the 0.3 value as a guideline, the Skitmore 2 bidder can now be seen to have clear periods of highly competitive bidding, with several instances of large amounts of 'money left on the table', and 11 uncompetitive bids (e.g., around contract 80). Also, the Skitmore 3 bidder can now be seen to really only have a few 'wild' bids, one of which wins the contract with nearly 25% money left on the table! Likewise, the USA Govt Agency bidder also has two 'wild' uncompetitive bids prior to a winning bid, while the Shaffer and Micheau bidder has four 'wild' uncompetitive bids, three of which are grouped together, again in the likely period of transition for that firm. The Hong Kong bidder records nine 'wild' uncompetitive bids, several of which are quite close to each other.

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and has been demonstrated empirically (eg., Flanagan & Norman 1984; Skitmore 1991; Drew &

All in all, the general impression here is that the 'wild' uncompetitive bids are not entirely random and it is possible that they may be 'cover prices'[3]. To see the extent to which the bidders are gradually converging on the winning bid, Table 2 gives the equivalent counts of the 'deficit' to the cusum situation. These can be tested for significance by a chi-square test (see Appendix for details). This shows no statistically significant trend to exist.

As an alternative, a one-way analysis of variance (ANOVA) can be used for this test. This is likely to be more powerful as it uses more of the information available. Table 3 summarises the results. This includes two additional datasets (1) Benjamin's (1969) series of 130 USA building contracts bid by his contractor over the period 1965-68 and (2) McCaffer and Pettitt's (1976) first Belgian contractor's series of 78 bids for public works contracts over the period 1971-74. The Table shows the mean and standard deviation of the 'deficit' values of the bid preceding a winning bid, the number of these bids (N) and the associated ANOVA p value. For example, the Skitmore 1 bidder has won 6 contracts (level 0) with a mean deficit bid of –0016272 (0.010646 standard deviation). On each of these six occasions, the deficit value of the bidder's preceding bid (level 1) had a mean of 0.099582 (0.088774 standard deviation). On one of these six occasions, the second last bid was a winning bid also, leaving five second last bids for analysis. These five second last bids had a mean deficit value of 0.049248 (0.24781 standard deviation), etc. The ANOVA tests for a significant difference between the mean deficit value of the last bid and the mean deficit value of the second last bid. In

Skitmore 1997).

[3] Defined as "… where a bidder enters a bid the value of which is advised by a competitor" (Skitmore

other words, the probability that 0.099582 and 0.049248 are from the same population. In this case, the probability is 0.254 which, although less than even chance, are not significant at the conventional 0.05 cut-off level. Similarly, the probability that the last, second last and third last mean deficit values are from the same population is 0.540, so again the differences are not significant. Looking through the results, it is clear that there are no significant differences, even at the 0.20 cut-off level.

Fig 3 shows the result of pooling the deficit values for (a) each and every level and (b) last and second last levels, and fitting a 3-degree polynomial regression equation. In both cases, the regression curve is shown to be virtually horizontal, again confirming any lack of trend with the data.

Discussion

The deficit statistic is rather more helpful than the cusum statistic in both observing and analysing bidding trends. In particular, it is shown that 'wild' bids do seem to group together on occasions. McCaffer and Pettitt's second observation, however, that winning bids are preceded by increasingly more competitive bids, is not supported.

ANALYSIS OF DEFICIT SUBSERIES

1989:175)

To examine McCaffer and Pettitt's first observation further, i.e., that there may be two types of bidding behaviour involved, the data for each case were divided into sub series and the sub series' means checked for significance by one-way ANOVA. This was done exhaustively for m=2, 3 and 4 sub series. For example, the Skitmore 1 contractor has a main series of 51 deficit values. We first divide these into two sub series groups, one containing two values and one containing the remaining 49 values. The means of each sub series was then calculated and the one-way ANOVA p value found. Another two sub series were then formed, with the first containing three values and the other containing the remaining 48 values. The one-way ANOVA p was again found. This was repeated until one-way ANOVA p values had been obtained for all possible pairs of sub series. The pair of sub series with the lowest p value was then noted. This was 5.28e-02 and occurred when the first and second sub series contained 25 and 26 values. These had a mean of 0.057 (0.057 sd) and 0.128 (0.170) respectively. The process was then repeated for all possible sub series triples. This time the lowest p value was found to be 2.06e-06, for the first, second and third sub series containing 37, 2 and 12 values. The means of these sub series are 0.073 (0.070 sd), 0.511 (0.492) and 0.086 (0.088 sd) respectively. Table 4 summarises the results for all the datasets.

As the method of selecting the sub series is non-random, the one-way ANOVA p values overestimate the true probabilities involved. To gain a first approximation of the correct probabilities, a Monte Carlo simulation was used. This involved generating simulated random values from a normal distribution with the same two first moments as the actual deficit values. For the Skitmore 1 data, therefore, 51 simulated deficit values were generated and subjected to the same analysis as above. This was repeated 100 times to obtain a distribution of lowest p values and the 5^{th} lowest value was then chosen to represent the fifth percentile. These simulated five percent cut-off value are also shown in Table 4 for comparison with the lowest p value recorded from the actual data. Naturally, in the event of the lowest p value for the actual data being less than the lowest p value from the simulated data, the difference between the associated sub series means was regarded as being significant. Table 4 shows these highlighted in **bold** text.

In examining Table 4, it is now clear that there are significant differences in the mean deficit values of the sub series' of six of the eight datasets. In several cases, these seem to be due to just a few unusual, or wild, bids. Rather less frequently, as with McCaffer and Pettitt's bidder, there are several quite lengthy adjacent sub series involved. Also, the midpoint break in Shaffer and Micheau's bidder is clearly identified.

Outliers

Table 5 summarises the effects of removing outliers. Of the six bidders affected, the removal of the single most top deficit value for three bidders, Skitmore 1, USA Govt and Shaffer and Micheau, is sufficient to render the means of the resulting best sub series statistically insignificant. Removal of the top two deficit values had the same effect for the McCaffer & Pettitt bidder. For the Hong Kong bidder, a group of four of the highest deficit values occurred between two winning bids – indicating a very short period of highly uncompetitive bidding. Removing these, together with a single outlier from elsewhere in the full series, again made the sub series' mean values statistically insignificant. Finally, for the Skitmore 2 bidder, a lengthy sub series of 15 deficit values were found with a significantly high mean value. In fact, this sub series (bids 20-34)

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contained five of the six highest deficit values in the whole series. This same bidder also recorded four successive wins - making yet another significant sub series. This series was therefore the most difficult to homogenise – requiring the removal of ten bids to avoid the best sub series of deficit values having significantly different means.

CONCLUSION

In testing the stationarity assumption, i.e., that individual bidding behaviour does not change over time, we have concentrated here on the two behaviour types proposed by McCaffer and Pettitt, i.e., that (1) winning bids come after a downward drift and (2) companies consistently bid low for some periods and then high in other periods. In doing this we have looked at the series of bidding data for six individual bidders, where all the bids for each contract are known, together with the identity of the associated bidders. This has been supplemented, where possible, with two further series of bids from individual bidders where only the value of the bidder's bid and lowest bid is known. The series' come from a variety of time periods, ranging from mid 1960s to 1990s, and countries, comprising UK (3), USA (3), Belgium (1) and Hong Kong (1). Of these, only two (Skitmore 1 and Benjamin) are a 'pure' series, having been collected from individual contractors – the remainder are for the most frequent bidders in a larger set of data obtained from client/owners or bidding agencies and therefore not guaranteed to be entirely complete.

In considering behaviour type (1), it was shown that McCaffer and Pettitt's cusum method is not very informative, especially where the bidder is relatively competitive.

A simpler and more informative approach was developed here using McCaffer's 'deficit' values, which is just the percentage the bid is above the lowest bid divided by 100. By comparing the bidder's mean deficit values for the ultimate bids immediately prior to the winning bid with the bidder's mean deficit values for penultimate bids, etc., it is shown that there is no significant downward drift for any of the individual contractors examined.

For behaviour type (2), we have developed the deficit analysis further to test all possible sub series of m=2,3,4 values. This has shown that sub series' with significantly different mean values do exist for six of the eight bidders analysed. In examining this further though, it is shown that, for three of these six bidders, the significant differences can be removed by simply eliminating the highest deficit value in their whole series. For another bidder, the same effect was achieved by eliminating the two highest values. Of the two remaining bidders, one was found to have a group of four of the highest deficit values occurring between two winning bids – indicating a very short period of very highly uncompetitive bidding. Removing these, together with a single outlier from elsewhere in the full series, again made the difference between the sub series' mean values statistically insignificant. For the remaining bidder, a lengthy sub series of 15 deficit values were found with a significantly high mean value. In fact, this sub series (bids 20-34) contained five of the six highest deficit values in the bidder's whole series. This same bidder also recorded four successive wins, making yet another significant sub series. This series was therefore the most difficult to homogenise – requiring the removal of ten bids to avoid the best sub series of deficit values having significantly different means.

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For the data used, the tests indicate that both type (1) and (2) behaviours do not occur frequently for construction contract bidders. For further testing, it may be possible to utilise some of the more standard treatments in, say, ARIMA modelling to identify specific types of trends. An alternative would be to follow McCaffer's (1976) approach in examining, for example, general departures from randomness by autocorrelation tests.

As Runeson & Skitmore (1999) have commented, stationarity is a vital assumption in the decision theoretic approach to bidding and serious violations of this assumption will be fatal to the approach as it is currently structured. What is less clear is the point at which such violations can be regarded as 'serious'. The existence of non-stationarity is not a new issue in the general literature. Economic time series, for example, as Ploberger and Phillips (2001:169) observe, "are often non-stationary … and there is good reason to believe that the trending mechanism is stochastic. However, [for econometric analysis] the precise form of the non-stationarity is not so much of an issue." Whether this will be true also for bidding modelling has yet to be determined although, for the data used, the analysis presented in this paper suggests that this may well be the case in practice.

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APPENDIX

Chi square test for 'deficit' method

Skitmore 1 data

Expected frequency for m=1 is n/2, m=2 is n/4, m$\geq$3 is n/4. So, for Skitmore 1 data, E_1= 6/2=3, E_2= 6/4=1.5, E_3= 6/4=1.5. That is

$$X^2 = \frac{(4-3)^2}{3} + \frac{(1-1.5)^2}{1.5} + \frac{(1-1.5)^2}{1.5} = 1/3 + \frac{1/4}{1.5} + \frac{1/4}{1.5} = 0..6667$$

This value of X^2 is not significant at the 5% level, when tested as a $\chi^2_{(2)}$ variable (for p<0.05, $\chi^2_{(2)} > 5.99$).

Skitmore 2 data

Ignoring m=0, $X^2 = \frac{(4-4.5)^2}{4.5} + \frac{(3-2.25)^2}{2.25} + \frac{(2-2.25)^2}{2.25} = 0..3333$, which again is not significant.

Skitmore 3 data

$X^2 = \frac{(2-2.5)^2}{2.5} + \frac{(2-1.25)^2}{1.25} + \frac{(1-1.25)^2}{1.25} = 2.6667$ which is again not significant.

USA Govt data

$X^2 = \frac{(4-3)^2}{3} + \frac{(1-1.5)^2}{1.5} + \frac{(1-1.5)^2}{1.5} = 0.6$ (not significant)

Shaffer and Micheau data

$X^2 = \frac{(4-5)^2}{5} + \frac{(2-2.5)^2}{2.5} + \frac{(2-1.25)^2}{1.25} + \frac{(2-1.25)^2}{1.25} = 1.2$ (not significant, for p<0.05, $\chi^2_{(3)} > 7.81$).

Hong Kong data

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$$X^2 = \frac{(5-4.5)^2}{4.5} + \frac{(2-2.25)^2}{2.25} + \frac{(1-1.125)^2}{1.125} + \frac{(1-1.125)^2}{1.125} = 0.1 \text{ (not significant).}$$

CAPTIONS TO FIGURES AND TABLES

Fig	**Caption**
1	Cusum analysis
2	Deficit analysis
3	Pooled deficit results

Table	**Caption**
1	Cusum bidding data
2	Previous falling bids (cumulative value in brackets)
3	Deficit analysis ANOVA results
4	Deficit grouping analysis
5	Outliers

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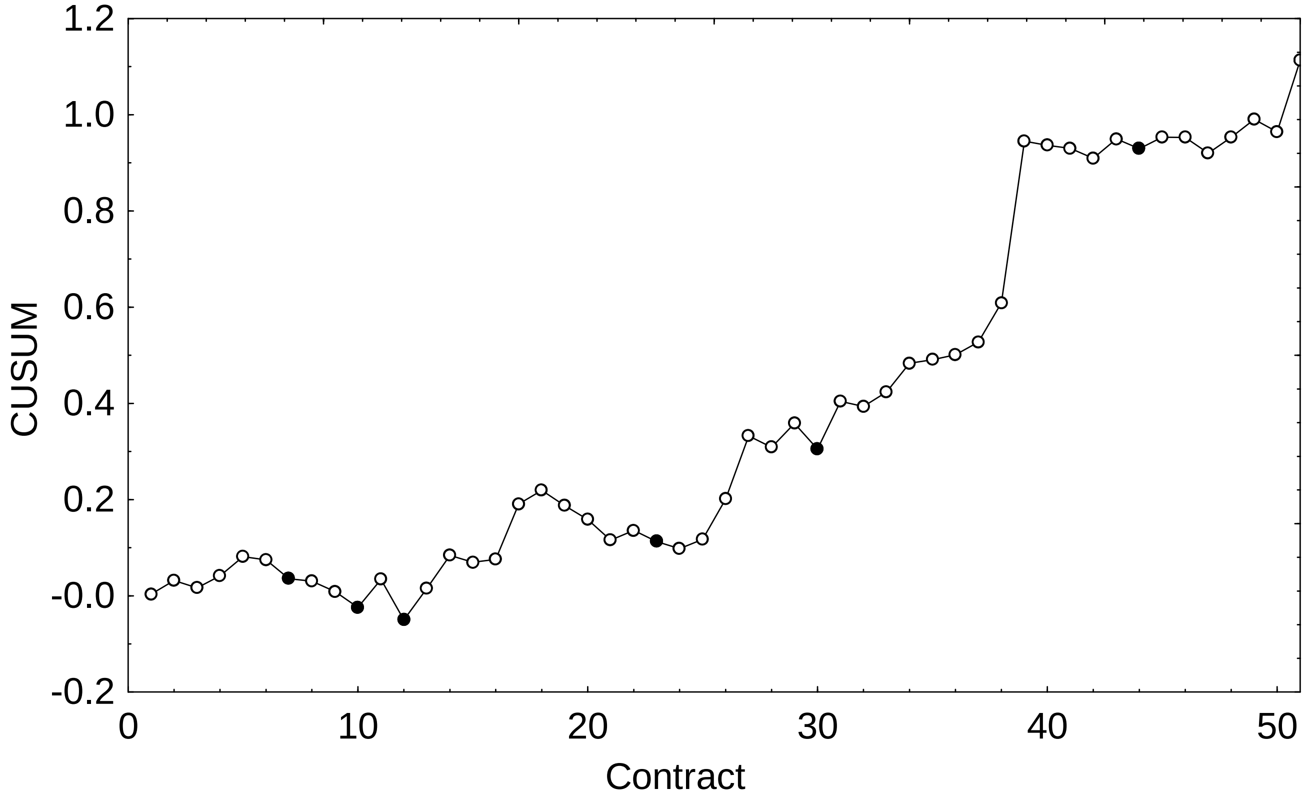
Fig 1a: Skitmore 1



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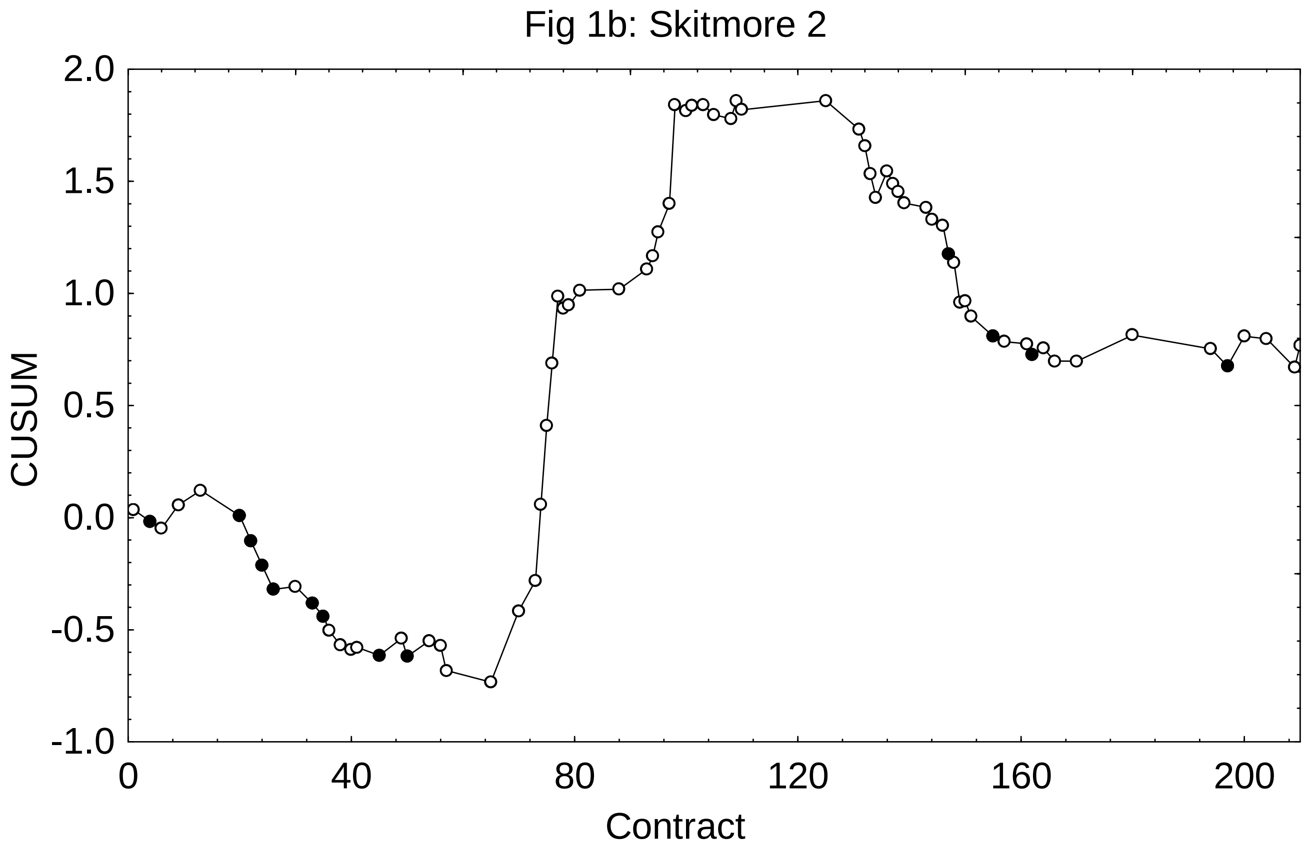

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Fig 1c: Skitmore 3

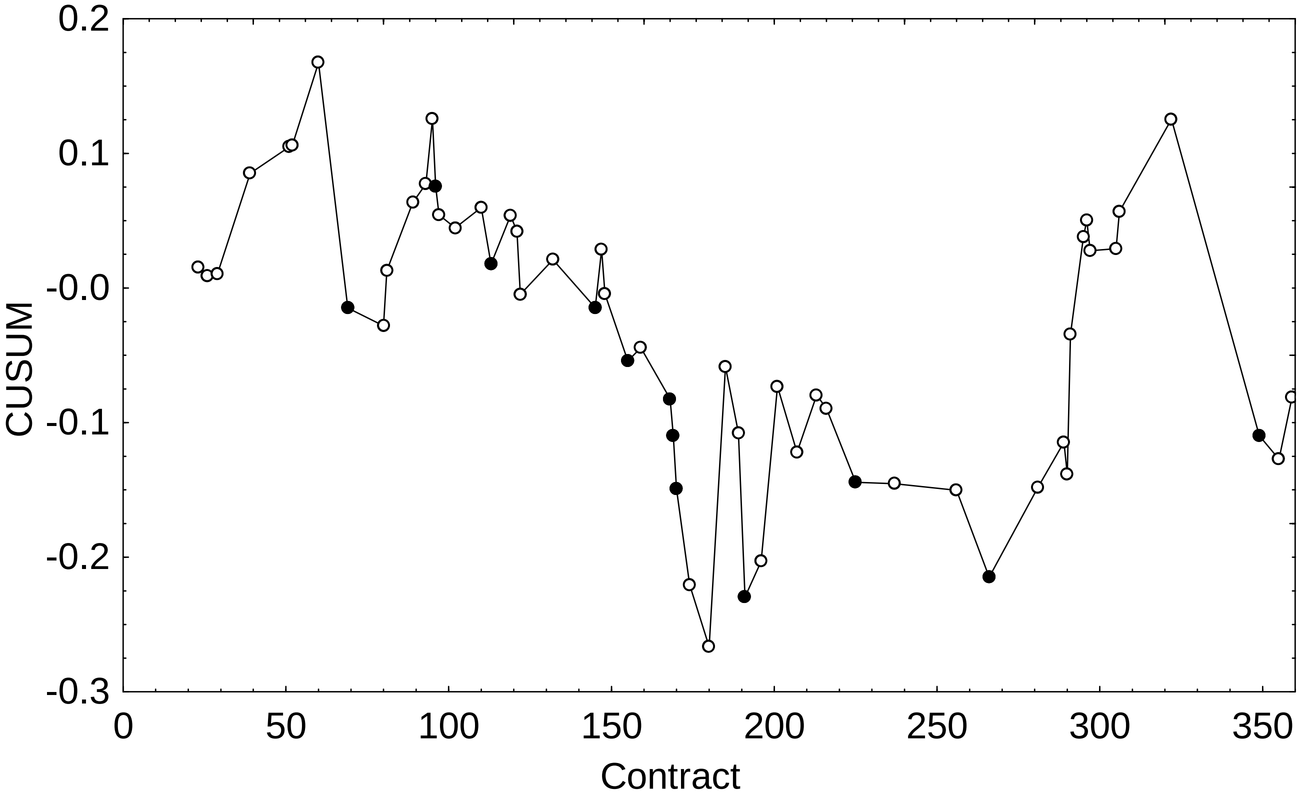
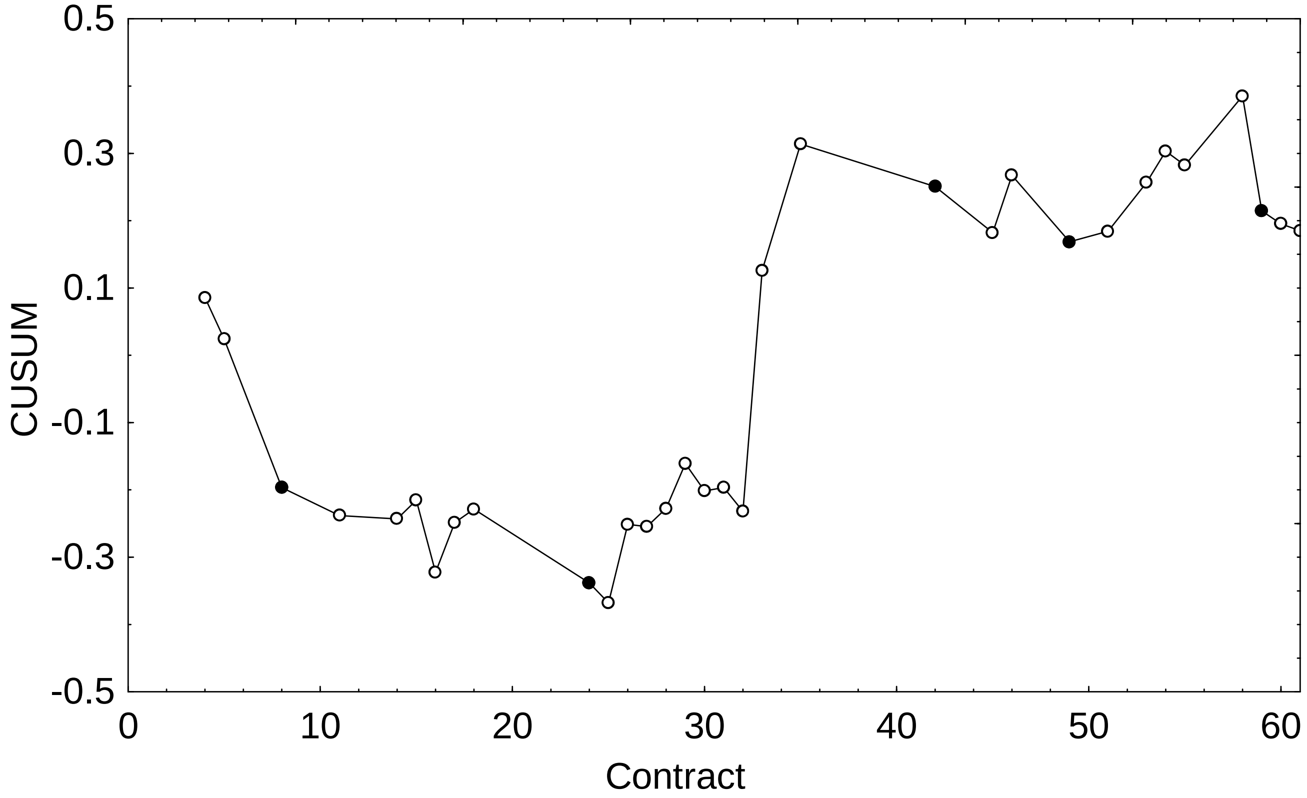


Fig 1d: USA Govt Agency



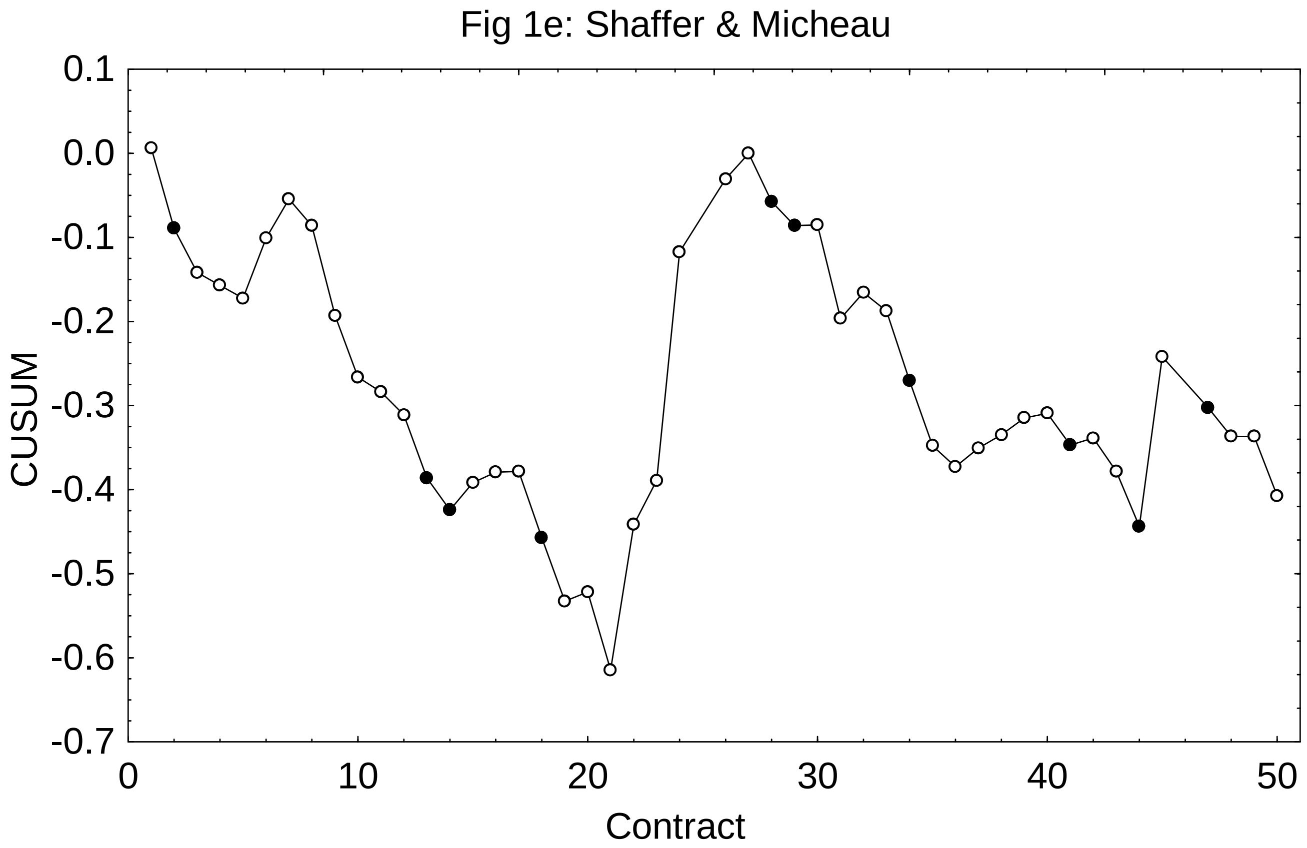


Fig 1e: Shaffer & Micheau
CUSUM
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
10
20
30
40
50
Contract

Fig 1f: Hong Kong data

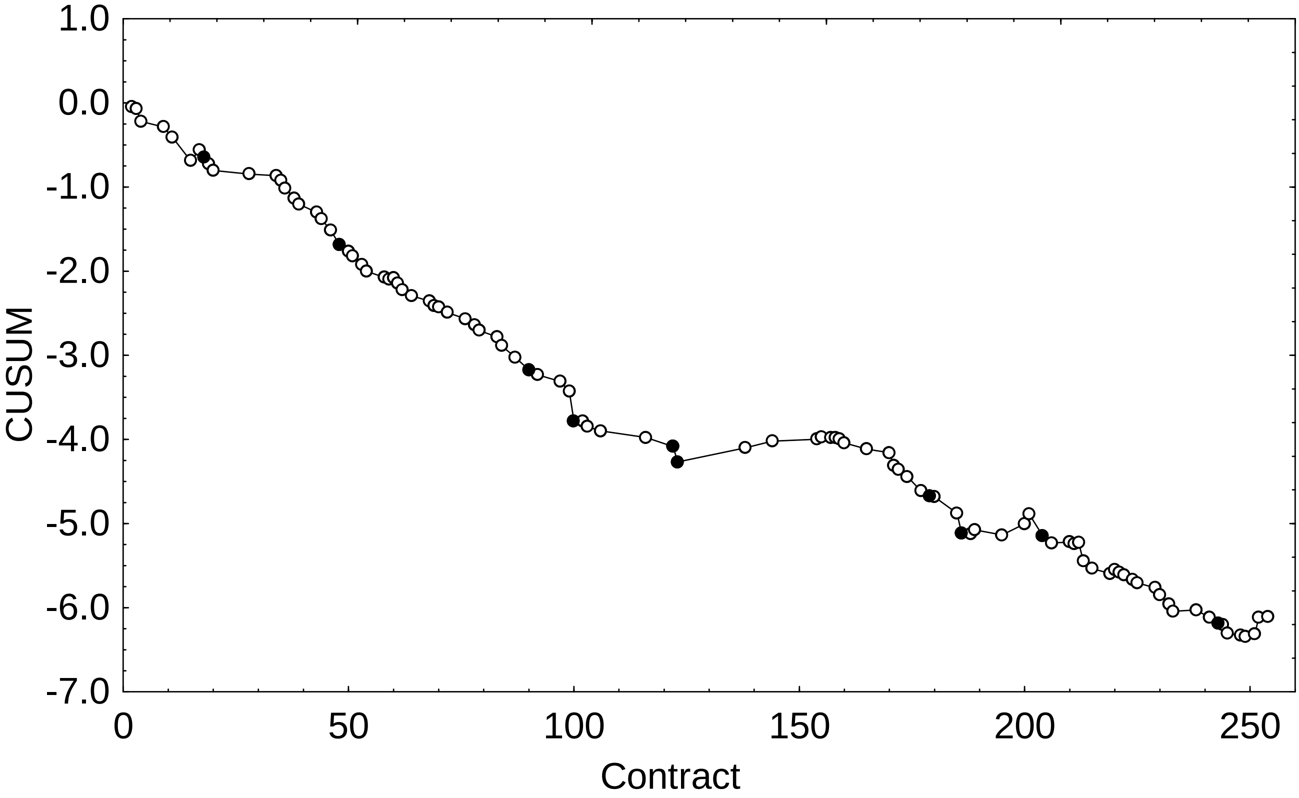


Fig 2a: Skitmore 1

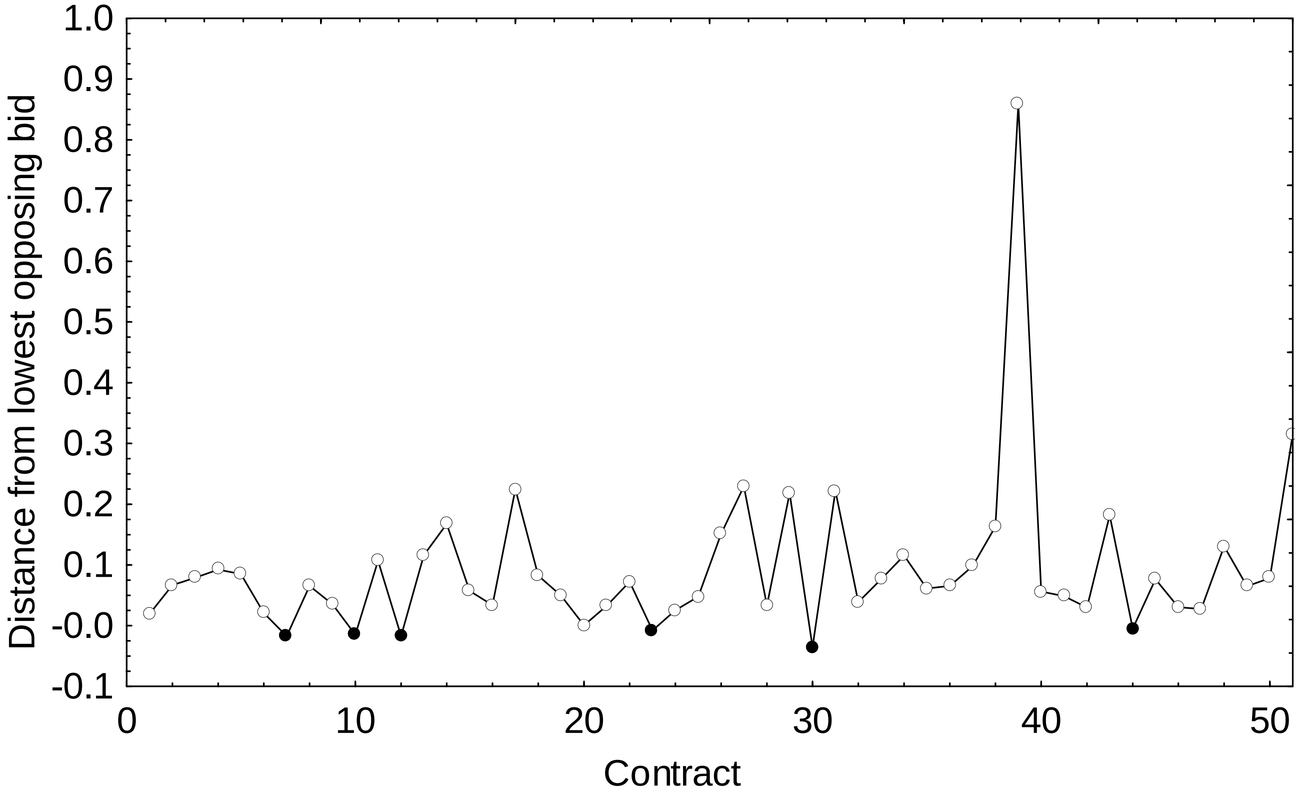


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Fig 2b: Skitmore 2

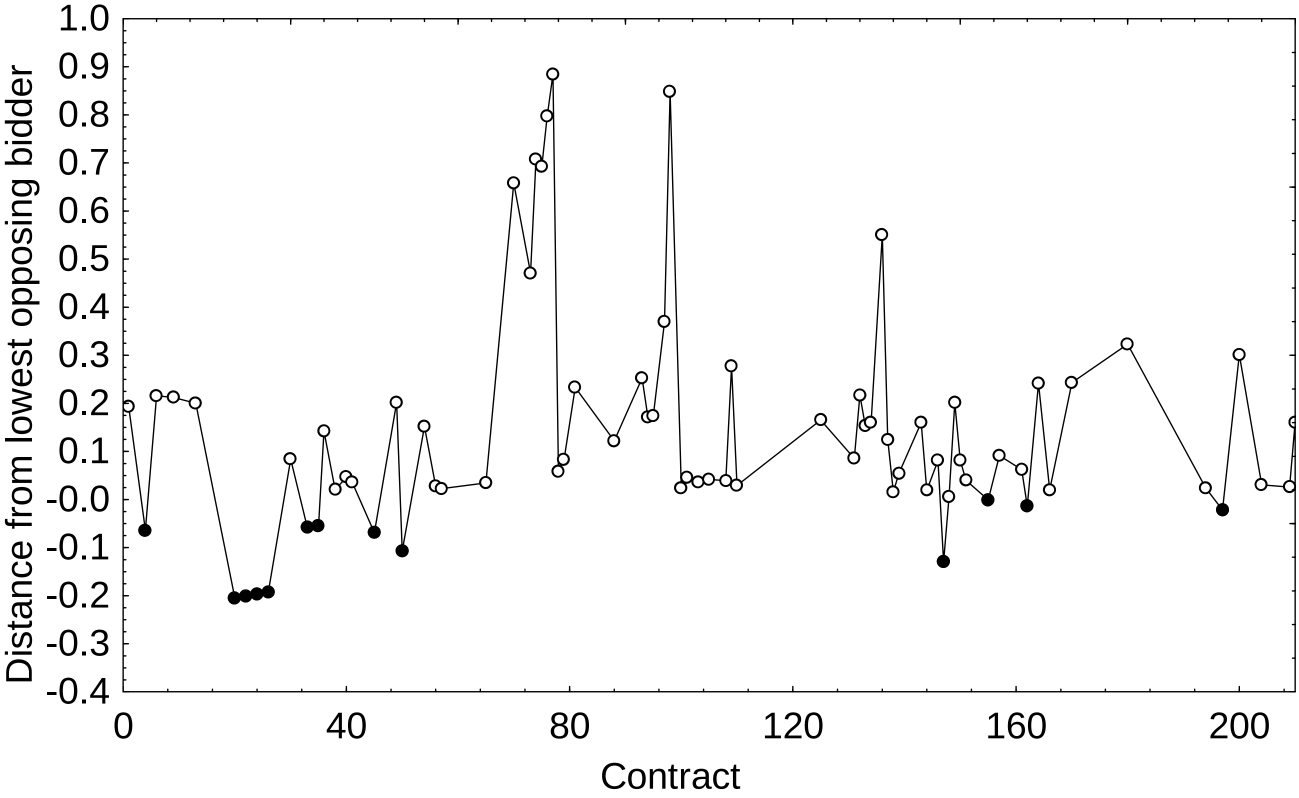
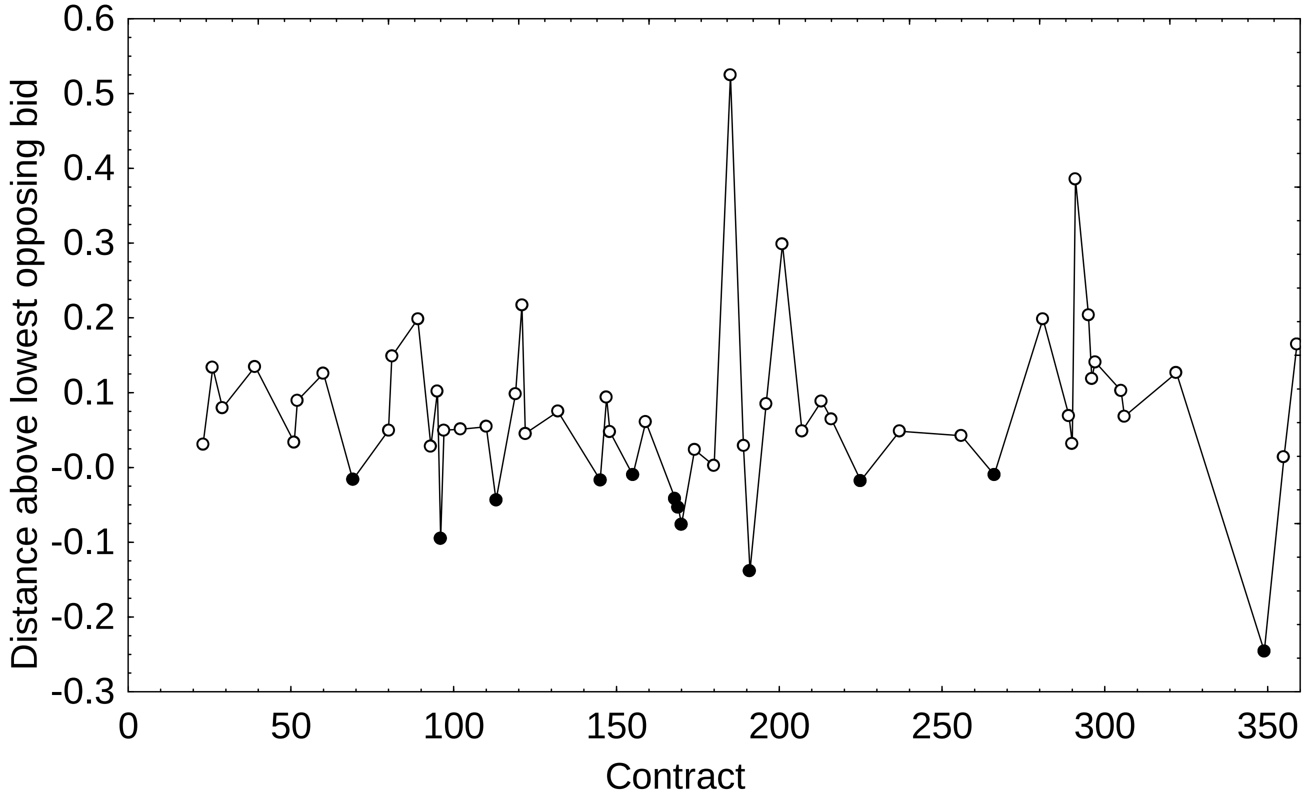


Fig 2c: Skitmore 3



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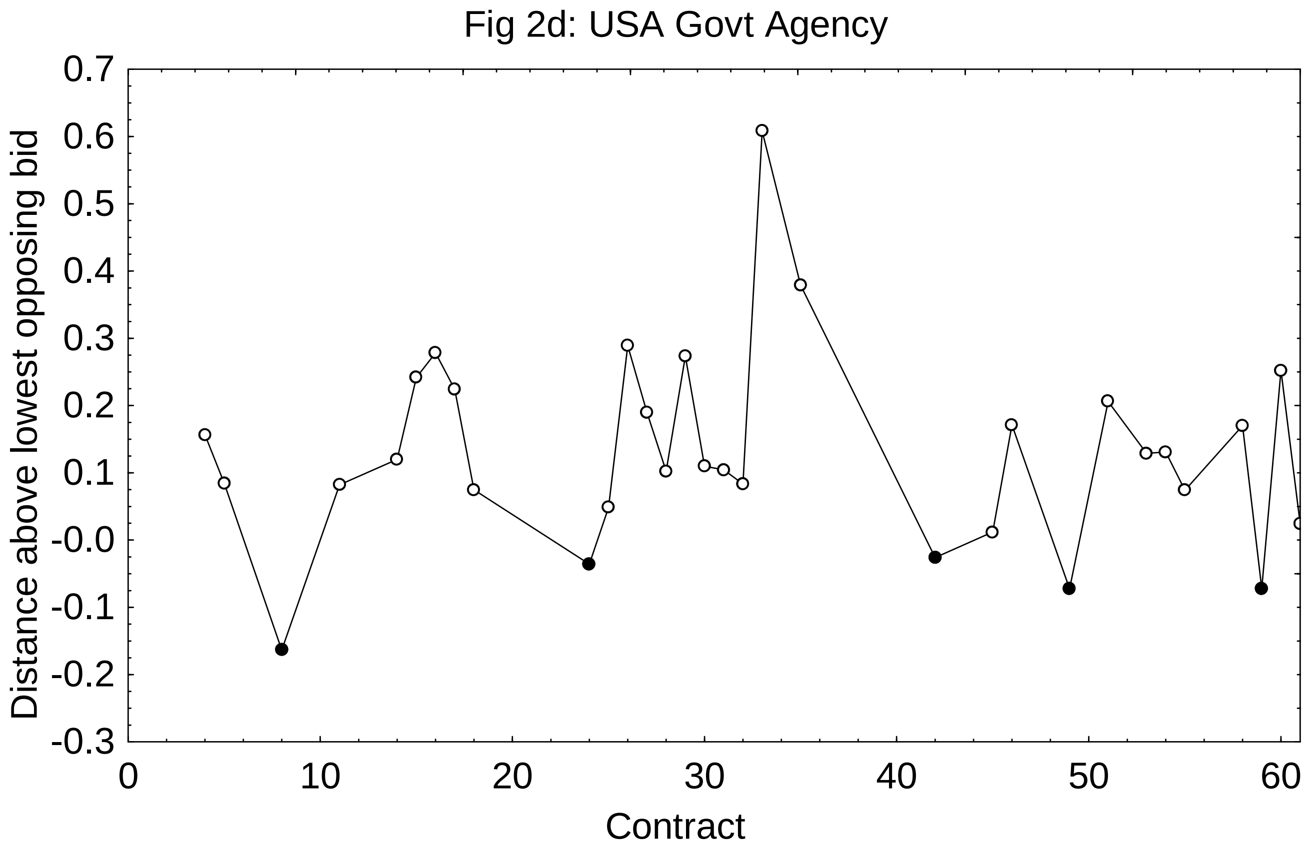
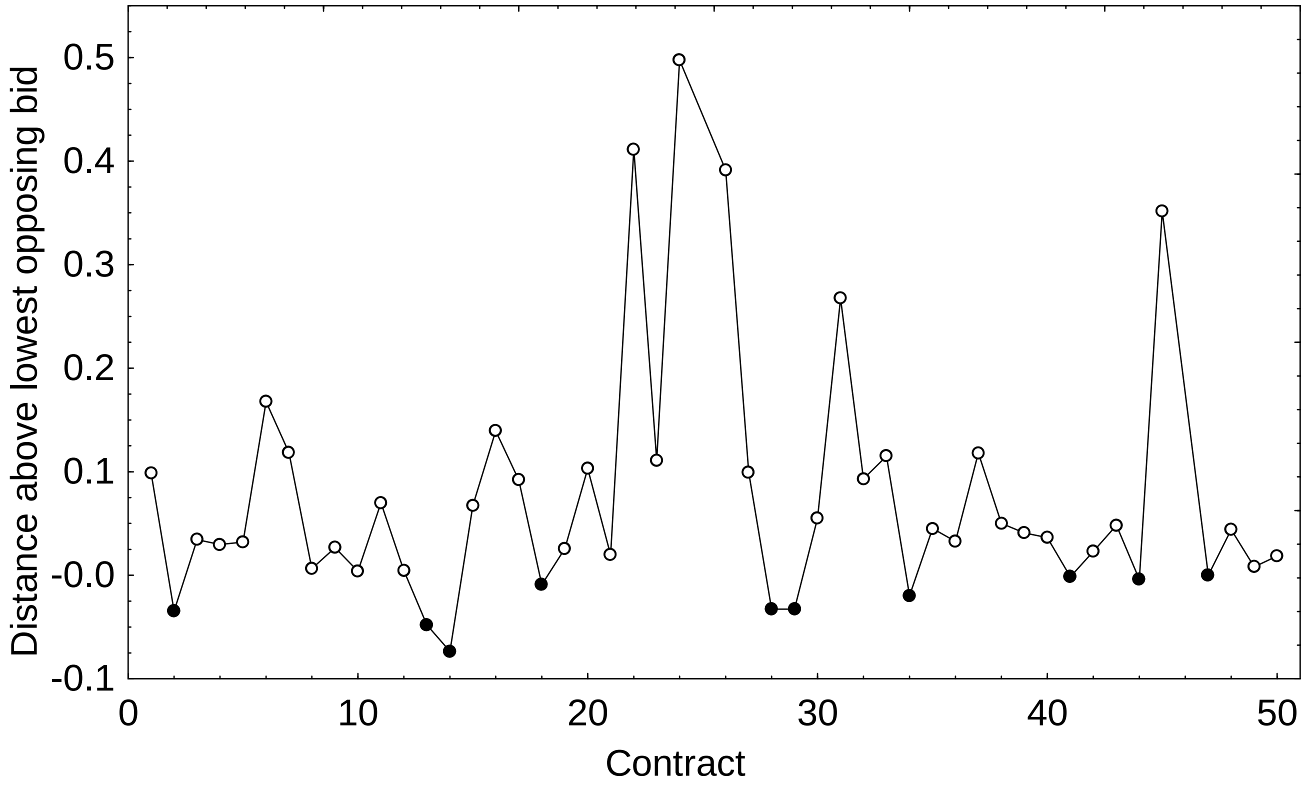


Fig 2d: USA Govt Agency
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.0
-0.1
-0.2
-0.3
Distance above lowest opposing bid
0
10
20
30
40
50
60
Contract

Fig 2e: Shaffer & Micheau



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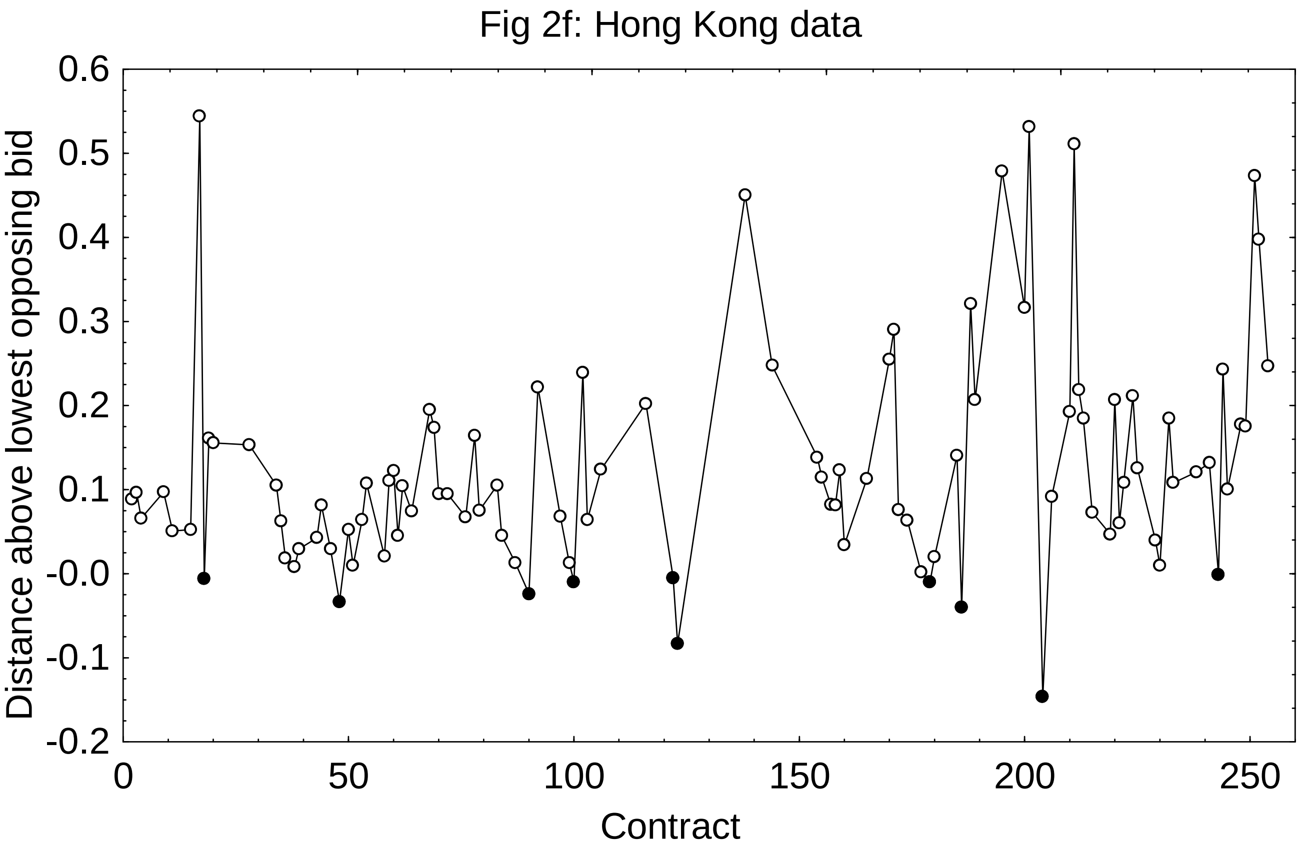

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Fig 3a: Pooled deficit values

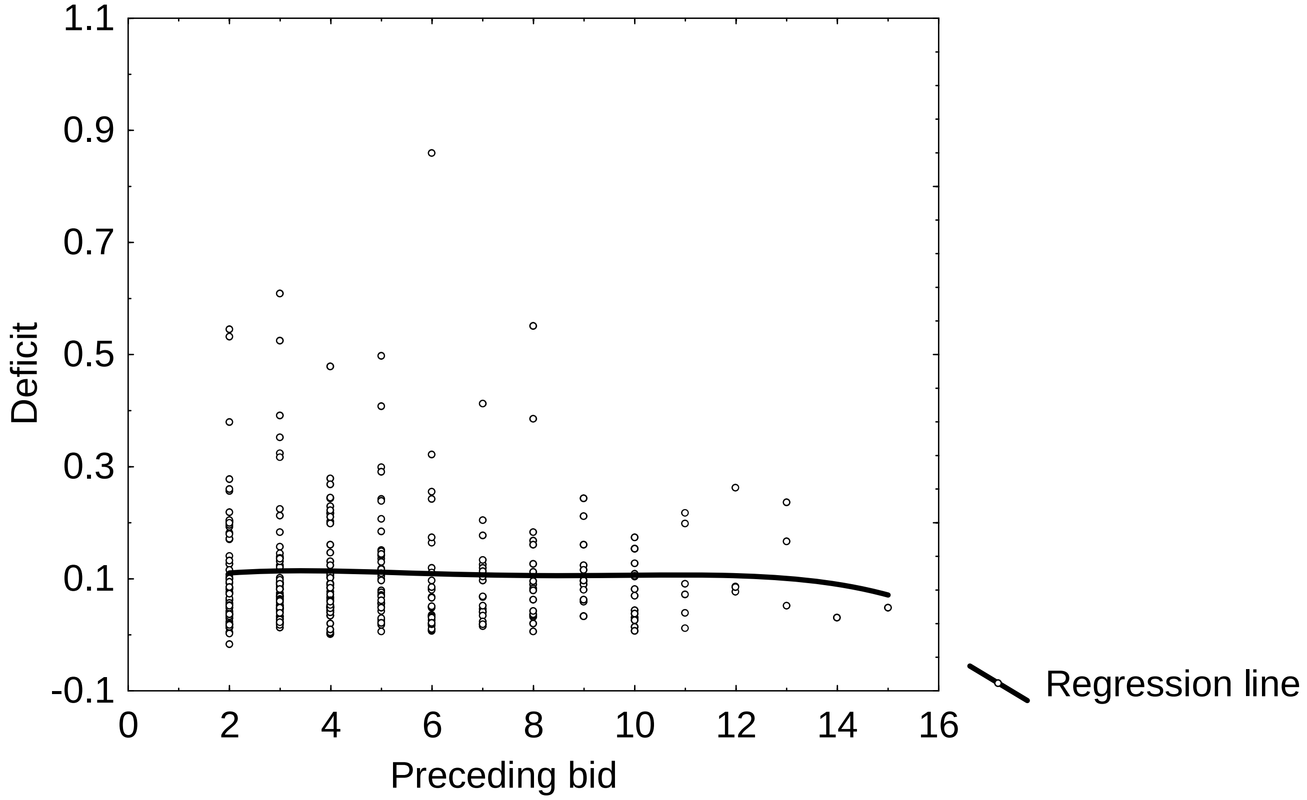
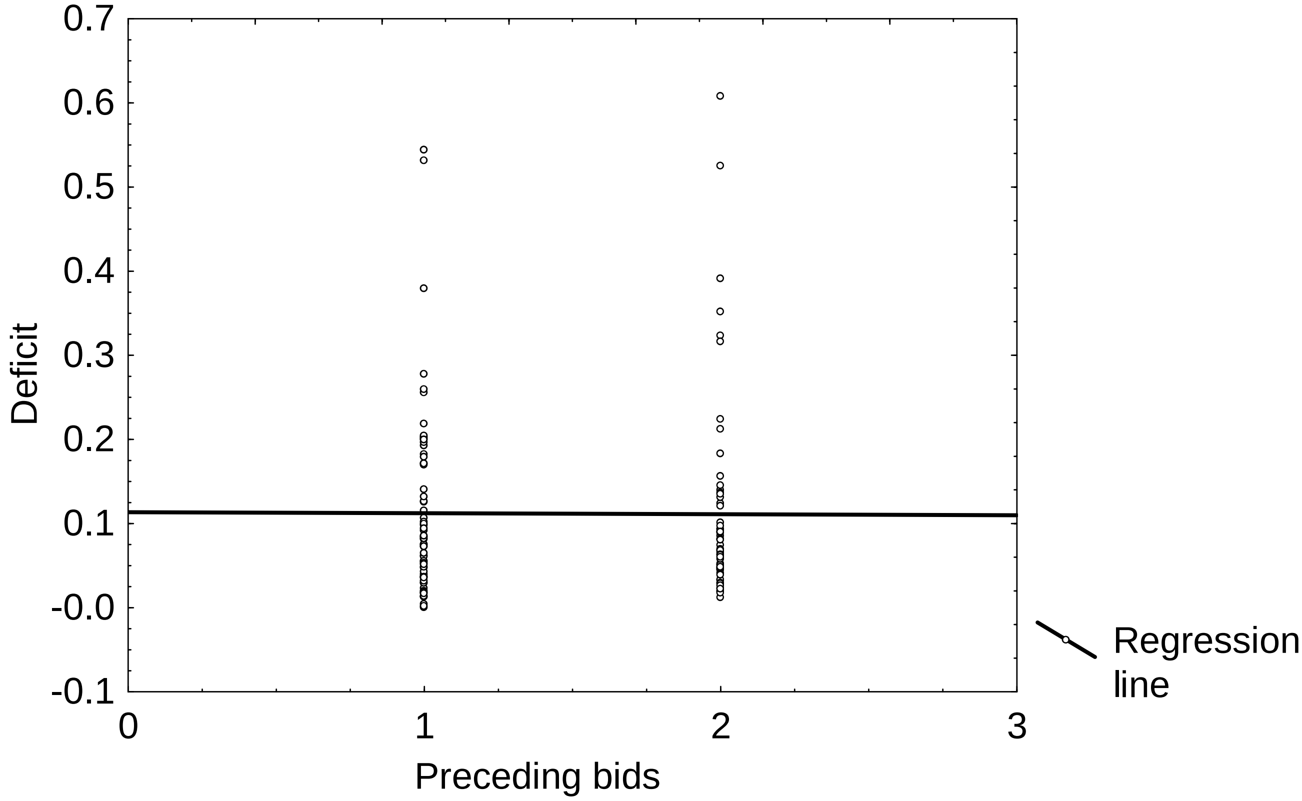


Fig 3b: Pooled deficit analysis for last and second last bids



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Data set	Source	Type	Period	No of contracts
1	Skitmore (1986)	London building contracts	1981-82	51
2	Skitmore (1986)	North of England public works contracts	1979-82	76
3	Skitmore (1986)	London building contracts	1976-77	57
4	Brown (1986)	USA Govt agency building contracts	1976-84	32
5	Shaffer & Micheau (1987)	USA building contracts	1965-69	48
6	Drew (2000)	Hong Kong Govt building contracts	1990-96	101

Table 1: Cusum bidding data

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Method		Skitmore 1		Skitmore 2		Skitmore 3		USA Govt Agency		Shaffer & Micheau		Hong Kong	
		No	%	No	%	No	%	No	%	No	%	No	%
CUSUM	m=1	4 (6)	100	5 (13)	100	6 (12)	100	4 (5)	100	5 (10)	100		
	m=2	1 (2)	33	4 (8)	62	4 (6)	50	1 (1)	20	3 (5)	50		
	m=3	- (1)	17	1 (4)	31	1 (2)	17	-	0	- (2)	20		
	m=4	- (1)	17	1 (3)	23	- (1)	8	-	0	- (2)	20		
	m=5	1 (1)	17	1 (2)	15	1 (1)	8	-	0	- (2)	20		
	m=6	-	0	- (1)	8	-	0	-	0	1 (2)	20		
	m=7	-	0	1 (1)	8	-	0	-	0	1 (1)	10		
DEFICIT	m=0	-	-	4 (13)	100	-	-	-	-	-		-	
	m=1	4 (6)	100	4 (9)	69	6 (12)	100	2 (5)	100	4 (10)	100	5 (9)	100
	m=2	1 (2)	33	3 (5)	38	5 (6)	50	2 (3)	60	2 (6)	60	2 (4)	44
	m=3	1 (1)	17	2 (2)	15	1 (1)	8	1 (1)	20	2 (4)	40	1 (2)	22
	m=4	-	0	-	0	-	0	-	0	2 (2)	20	1 (1)	11

Table 2: Previous falling bids (cumulative value in brackets)

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Preceding bid	N	Means	Std.Dev.	ANOVA p
Skitmore 1				
0	6	-.016272	.010646	-
1	6	.099582	.088774	-
2	5	.049248	.024781	0.254
3	4	.092763	.098434	0.540
4	4	.084060	.046671	0.690
5	4	.263988	.396922	0.452
Skitmore 2				
0	13	-.101176	.076207	-
1	9	.102396	.074512	-
2	6	.128972	.115907	0.595
3	5	.168090	.087943	0.453
4	4	.055340	.061236	0.289
5	2	.128885	.160054	0.473
6	1	.124520	-	-
7	1	.550140	-	-
8	1	.159690	-	-
9	1	.152850	-	-
10	1	.217330	-	-
11	1	.085680	-	-
12	1	.165620	-	-
Skitmore 3				
0	12	-.063744	.069521	
1	10	.072842	.034219	
2	9	.115379	.155273	0.409
3	7	.093180	.083934	0.674
4	6	.140853	.090017	0.594
5	4	.083270	.028453	0.691
6	2	.168755	.050084	0.658
7	2	.208220	.250570	0.585
8	1	.032210	-	-
9	1	.069420	-	-
10	1	.198550	-	-
USA Govt				
0	5	-.073898	.054214	-
1	5	.175986	.122355	-
2	5	.214958	.234108	0.750
3	3	.164153	.101610	0.905
4	3	.158390	.073169	0.955

Preceding bid	N	Means	Std.Dev.	ANOVA p
Shaffer & Micheau				
0	10	-.025581	.023543	-
1	8	.083171	.052002	-
2	7	.158450	.150639	0.206
3	5	.087442	.103315	0.367
4	4	.174403	.218783	0.536
5	3	.050020	.054416	0.526
6	3	.191670	.193850	0.576
7	2	.093585	.104645	0.674
8	2	.067555	.050169	0.720
9	2	.027765	.002906	0.693
Hong Kong				
0	10	-.035832	.046077	-
1	9	.178650	.215341	-
2	9	.099113	.088083	0.320
3	8	.143379	.146713	0.577
4	7	.160184	.095010	0.708
5	6	.137323	.131523	0.832
6	5	.067162	.039061	0.707
7	5	.081146	.034686	0.780
8	4	.133668	.053084	0.750
9	4	.129183	.041837	0.806
Benjamin				
0	20	-.025442	.031367	-
1	17	.102320	.095253	-
2	15	.074585	.048894	0.318
3	14	.089888	.069384	0.583
4	12	.096785	.104871	0.804
5	10	.050117	.048901	0.494
6	9	.058944	.052380	0.473
7	8	.087341	.061410	0.575
8	7	.106621	.063474	0.593
9	7	.062506	.048593	0.606
10	4	.052985	.035324	0.616
11	3	.141107	.105188	0.535
12	2	.143720	.130094	0.522
13	1	.030120	-	-
14	1	.048230	-	-
McCaffer& Pettitt				
0	4	-.020320	.011261	-
1	4	.120218	.085000	-
2	4	.103058	.038489	0.726
3	4	.071488	.018707	0.475

Table 3: Deficit analysis ANOVA results

Case	m	F	df1	df2	p	Sim p	N	mean	sd
Skitmore 1	2	3.94	1	49	5.28e-02	5.45e-03	25	0.057	0.057
							26	0.128	0.170
	3	17.42	2	48	2.06e-06	1.36e-04	**37**	**0.073**	0.070
							2	**0.511**	0.492
							12	**0.086**	0.088
	4	13.05	3	47	2.51e-06	8.21e-05	**37**	**0.073**	0.070
							2	**0.511**	0.492
							10	**0.064**	0.055
							2	**0.197**	0.168
Skitmore 2	2	10.93	1	74	1.46e-03	2.04e-03	**23**	**0.019**	0.142
							53	**0.198**	0.241
	3	46.54	2	73	9.31e-14	1.08e-04	**23**	**0.019**	0.142
							6	**0.702**	0.140
							47	**0.134**	0.162
	4	36.45	3	72	1.93e-14	3.40e-05	**23**	**0.019**	0.142
							6	**0.702**	0.140
							9	**0.257**	0.241
							38	**0.104**	0.125
Skitmore 3	2	2.35	1	55	1.31e-01	3.50e-03	32	0.051	0.074
							25	0.098	0.153
	3	4.24	2	54	1.95e-02	1.43e-04	47	0.064	0.106
							2	0.295	0.128
							8	0.061	0.133
	4	3.93	3	53	1.32e-02	1.15e-04	32	0.051	0.074
							2	0.277	0.351
							20	0.098	0.113
							3	-0.022	0.208
USA Govt	2	2.26	1	30	1.43e-01	1.42e-03	20	0.163	0.161
							12	0.083	0.110
	3	11.00	2	29	2.79e-04	7.00e-04	**18**	**0.126**	0.115
							2	**0.494**	0.162
							12	**0.083**	0.110
	4	8.88	3	28	2.70e-04	1.17e-04	4	0.040	0.140
							14	0.150	0.099
							2	0.494	0.162
							12	0.083	0.110
Shaffer & Micheau	2	3.17	1	46	8.18e-02	2.27e-03	21	0.042	0.062
							27	0.103	0.147
	3	22.25	2	45	1.92e-07	4.96e-04	**21**	**0.042**	0.062
							4	**0.353**	0.168
							23	**0.059**	0.091
	4	17.68	3	44	1.13e-06	7.97e-05	**21**	**0.042**	0.062
							2	**0.261**	0.212
							2	**0.444**	0.075
							23	**0.059**	0.091
Hong Kong	2	13.86	1	99	3.28e-04	8.12e-04	**69**	**0.095**	0.102
							32	**0.192**	0.157
	3	15.05	2	98	2.00e-06	1.19e-04	**69**	**0.095**	0.102
							5	**0.371**	0.132
							27	**0.159**	0.140
	4	15.65	3	97	2.23e-08	3.44e-05	**69**	**0.095**	0.102

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							5 **24** **3**	**0.371** **0.132** **0.373**	0.132 0.199 0.115
Benjamin	2	8.25	1	128	4.77e-03	1.98e-03	2 12 8	0.220 0.064	0.264 0.073
	3	6.40	2	127	2.25e-03	8.53e-05	2 79 49	0.224 0.054 0.082	0.264 0.058 0.090
	4	7.30	3	126	1.49e-04	1.38e-05	2 10 6 2 20	0.220 0.062 0.249 0.061	0.264 0.066 0.019 0.086
McCaffer & Pettitt	2	12.80	1	76	6.08e-04	1.90e-03	**17** **61**	**0.043** **0.106**	0.036 0.069
	3	10.00	2	75	1.41e-04	1.47e-04	**30** **26** **22**	**0.061** **0.134** **0.087**	0.051 0.076 0.053
	4	8.60	3	74	5.71e-05	7.15e-05	**17** **19** **20** **22**	**0.044** **0.089** **0.143** **0.087**	0.036 0.065 0.076 0.053

Table 4: Deficit grouping analysis

Bidder	Highest deficit value	Number removed	Cut-off	m=2	m=3	m=4	Comment
Skitmore 1	0.859	1	Top value	1.90e-02	1.16e-2	5.05e-03	
Skitmore 2	0.885	10	-0.19 and 0.69	8.32e-02	1.28e-02	3.60e-02	5 top values in one group
Skitmore 3	0.525	0	NA				
USA Govt	0.608	1	Top value	1.73e-01	8.01e-02	7.29e-03	
Shaffer & Micheau	0.498	1	Top value	1.69e-01	5.85e-03	7.91e-03	
Hong Kong	0.544	5	One outlier	1.15e-02	1.70e-03	3.30e-04	4 high values in one group between wins
Benjamin	0.407	0	NA				
McCaffer & Pettitt	0.326	2	0.25	5.15e-04	5.13e-04	3.39e-04	

Table 5: Outliers

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