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Causality of price movements in VIX exchange-traded products and VIX futures contracts

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Abstract

Purpose – The authors analyse six actively traded VIX Exchange Traded Products (ETPs) including 1x long, −1x inverse and 2x leveraged products. The authors assess their impact on the VIX Futures index benchmark.
Design/methodology/approach – Long-run causal relations between daily price movements in ETPs and futures are established, and the impact of rebalancing activity of leveraged and inverse ETPs evidenced through causal relations in the last 30 min of daily trading.
Findings – High frequency lead lag relations are observed, demonstrating opportunities for arbitrage, although these tend to be short-lived and only material in times of market dislocation.
Originality/value – The causal relations between VXX and VIX Futures are well established with leads and lags generally found to be short-lived and arbitrage relations holding. The authors go further to capture 1x long, −1x inverse as well as 2x leveraged ETNs and the corresponding ETFs, to give a broad representation across the ETP market. The authors establish causal relations between inverse and leveraged products where causal relations are not yet documented.

Keywords Volatility, Causality, Exchange traded products
Paper type Research paper

1. Introduction

Whaley (1993) foresaw the importance of VIX derivatives a decade before their introduction in 2004. A suite of derivatives and related products has since emerged based on the CBOE Volatility Index (VIX). Exposures are in the form of VIX Exchange Traded Products (ETPs). This class includes Exchange Traded Funds (ETFs) which are asset- or futures-based, as well as Exchange Traded Notes (ETNs) which are unsecured notes promising the daily rate of return of a benchmark index with no direct claim on underlying assets. These products allow customised volatility exposure for a broad range of trading and hedging strategies. Markets for VIX derivatives have grown to rival SPX and SPY options markets for trading and hedging volatility. Figure 1 shows that the aggregate sensitivity of VIX Options and Futures to changes in SPX ("vega") has exceeded that of SPX and SPY options since late 2012.

Initially, institutional investors might have preferred to trade ETPs rather than the underlying futures contracts due to frictional costs, mandate restrictions, overheads and execution risks in rolling futures contracts. However, with spreads and trading commissions for ETPs now very low, the only material reasons remaining are convenience and mandate restrictions. As an asset class, ETPs have a broad appeal to retail investors over other

JEL Classification — C22, C32, G13, G14

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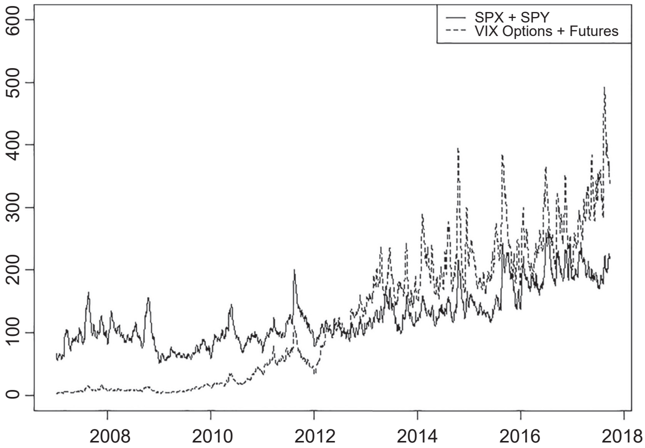


Note(s): This figure shows One Month Average Daily Gross Traded Vega ($'m). The exponentially weighted moving average of gross vega is shown for the two markets calculated using OptionMetrics data provided by UBS from January 2007 to February 2018. Calculations for VIX Options and Futures assume put-call parity to estimate forward and discount rates for each day and expiry. Black-Scholes implied volatility and vega is calculated based on VIX Futures prices rather than spot VIX. The models are not available beyond the liquidity event in February 2018 when XIV was recalled and SVXY/UVXY leverage changed

Source(s): Figure by authors

Figure 1.
Aggregate sensitivity of VIX Options and Futures to changes in SPX and SPY

investment vehicles (see Broman, 2016; Agapova, 2011). VIX ETPs have become an increasingly popular means to consider volatility as an asset class, to manage risk exposures and to access VIX derivatives markets that would otherwise have been unavailable to retail investors (Whaley, 2013). Whaley (2013) documents the perils of trading volatility using ETPs, notably the "contango trap". In this case, the short end of the futures price curve shows a steep upward slope for 80% of the time, except during short windows where the curve slopes downwards ("backwardation") (see also Shu and Zhang, 2012; Gehricke and Zhang, 2018; Johnson, 2017).

While there are many venues to trade and hedge market volatility, along with several potential determinants of price discovery and volatility, we would generally expect price discovery to occur in the deepest, most liquid market with lower trading costs (see Fleming *et al.*, 1996; Ozturk *et al.*, 2017; Benos and Sagade, 2016; Riordan and Storkenmaier, 2012; Chakravarty *et al.*, 2004; Chu *et al.*, 1999; Chung and Chuwonganant, 2018; Luo and Zhang, 2017; Andersen and Bondarenko, 2014a and b; Barinov and Wu, 2014; Barunik *et al.*, 2016). All else equal, we would expect VIX product prices to become more informative as more investors enter the market and help to set prices and eliminate arbitrage opportunities. VIX is not a tradable asset, and arbitrage is not possible except perhaps on expiry (see, e.g. Zhu and Lian, 2012; Lin, 2007; Zhang and Zhu, 2006). VIX Futures represent the forward expectation of the VIX. VIX Futures are the dominant market for trading and hedging volatility, involving no up-front costs, reflecting volatility first and predicting the direction of VIX ("tail wags dog") (see Bollen *et al.*, 2017; Frijns *et al.*, 2016; Zhang *et al.*, 2010; Shu and Zhang, 2012; Zhang and Zhu, 2006; Konstantinidi and Skiadopoulos, 2011; Lin, 2007; Dian-Xuan *et al.*, 2017; Chen and Tsai, 2017). VIX Options are linked to Futures through put-call parity and lead/lag relations which are short-lived (Bollen *et al.*, 2017).

VIX products are becoming increasingly important instruments to trade and hedge volatility, and there is a growing literature on causal relations between complex VIX ETPs and the VIX Futures from which they are most often constructed (see Frijns *et al.*, 2016; Shu and Zhang, 2012; Posselt, 2021). Prior studies have focused on relations between 1x long VIX ETP (VXX); VXX initially had an undue influence on VIX derivatives markets which dissipated as VIX Futures trading grew (see, e.g. Bordonado *et al.*, 2017; Gehricke and Zhang, 2018; and Simon and Campasano, 2014). There are some recent studies on inverse and leveraged products (see, e.g. Fernandez-Perez *et al.*, 2018; Whaley, 2013), but the causality between VIX, VIX ETPs and VIX Futures across different market environments remains an open question [1].

Here, we study a spectrum of ETNs, including three of the most actively traded ETNs. We also study the constant maturity VIX Futures index benchmark in our multivariate analysis of causality. VXX provides daily long exposure to a 30 days maturity VIX Futures index. TVIX has a daily two times leveraged exposure, and XIV has a daily inverse exposure. We include the corresponding ETFs (VIXY, UVXY and SVXY, respectively).

Causal relations between products are studied using methods which are well adapted to high-frequency and asynchronous data (e.g. Yamamoto and Kurozumi, 2006; Hayashi and Yoshida, 2005; Finucane, 1999). We hypothesise that like VIX Futures, VIX ETPs lead VIX (see Bollen *et al.*, 2017 and Kao *et al.*, 2018). We investigate the influence of term structure on lead–lag relations; traders operating in segmented markets may change their positions in response to expectations of volatility, as the futures curve moves from contango to backwardation [2].

Our findings have implications for studies of supply and demand and limitations to arbitrage. The causal relations we show evidence break-downs in arbitrage relations; contrary to economic theory we provide evidence that supply can be significantly inelastic in the short term. Given the high-frequency nature of the markets we study, we demonstrate and lead–lag relations between directly related VIX ETPs and VIX Futures, which can in some cases persist for time periods long enough that they could well-represent statistically significant arbitrage opportunities. A related and more applied future study could involve analysing the economic significance of these arbitrage opportunities and limits to liquidity.

This paper is structured as follows. VIX product data are described in Section 2. Section 3 details the methods and results. Finally, Section 4 concludes.

2. VIX product data

2.1 Data sources

We study the price series for the S&P 500 VIX Futures index against three VIX ETNs and three corresponding ETFs, each of which is benchmarked to S&P 500 VIX Futures indexes. We use intraday time series for all four series.

VIX Futures contracts are written on the VIX with a denomination of $1,000 times the index. Intraday trade and quote data and daily open interest for the two nearest maturity VIX Futures contracts were acquired from Thomson Reuters Tick History (TRTH), available through SIRCA (Securities Industry Research Centre of Asia–Pacific). Data were collected to the nearest millisecond.

Standard and Poor's has reported the S&P 500 VIX short-term total return index (SPVXSTR) since 20 December 2005. This comprises a weighted position in the two nearest maturity futures contracts, with positions rebalanced at the end of each day to maintain a 30 days maturity. The total return index (SPVXSTR) differs from the excess return index (SPVXSTER) in that it incorporates return on the collateralised futures position in the form of return on 3-month Treasury bills. SPVXSTR data were available in five second intervals from TRTH.

Daily NYSE trade and quote data (TAQ) was obtained for VXX, VIXY, XIV, SXVY, TVIX and UVYX from Wharton Research Data Services from the time of inception of each ETP until 31 March 2018. On 15 February 2018, Nasdaq Stock Market suspended trading of the XIV ETNs after the market close and instituted delisting proceedings. In addition to XIV's termination by Credit Suisse, ProShares decided to reduce the leverage factors on UVXY and SVXY taking effect 28 February 2018. Our sample ends February 15, capturing the liquidity event on February 5, but excluding the interventions of product elimination of XIV and leverage ratios changes of UVXY and SVXY. The iPath S&P 500 VIX Short-Term Futures ETN (VXX) was chosen for the study as one of two VIX ETPs launched on 29 January 2009. As a first mover, it remains one of the most actively traded ETPs with a market cap of over $1bn on 31 March 2018, despite losing over 99% of its value along with three reverse spits since inception. Its performance is benchmarked to the daily index return on SPVXSTR, less management fees and expenses. Institutions might be expected to use VXX to protect their portfolios at times of high volatility when the futures curve moves from contango to backwardation, while retail tends to have a buy-and-hold interest in VXX [3]. We also include the corresponding ProShares VIX Short-Term Futures ETF (VIXY) which has seen similar value destruction and reverse splits since its inception on 1 March 2011.

The Velocity Shares Daily Inverse VIX Short-Term exchange-traded note (XIV) was launched on 29 November 2010. XIV is effectively the opposite of VXX being an investment on contango. Institutions would logically tend to invest more in XIV and reduce their holdings in VXX at low volatility regimes. It was the second most active ETP, and had a very strong year, more than doubling in 2017, and benefitting from rolling-down a consistently steep futures curve at ~8.5% per month. Conversely, XIV will suffer accelerating losses when the market goes into backwardation. It has most likely had the highest institutional ownership for its tactical application (Whaley, 2013). When the VIX had an unprecedented 115% spike from 17 to 37% in a two-hour period on 5 February 2018, this was the largest percentage gain in VIX in any one day recorded. An "acceleration event" was triggered to avoid the value of the ETN going to zero [4]. XIV was benchmarked to the SPVXSTER futures index with a multiplier of -1, meaning that it tracked a short position in the futures index. Although XIV has ceased trading, similar products such as ProShares Short VIX Short-Term Futures ETF (SVXY) continue to trade, and we also include SVXY in our study since its inception on 3 October 2011.

The VelocityShares Daily 2x VIX Short-Term ETN (TVIX) is also amongst the most active ETPs. This was launched alongside the XIV on 29 November 2010 with a market cap of $0.5bn on 31 March 2018. TVIX is benchmarked to SPVXSTER with a multiplier of 2, meaning that it promises the daily return of index multiplied by 2 [5]. It is owned 99% by retail shareholders (Whaley, 2013). TVIX also suffered a liquidity event when Credit Suisse stopped issuing new shares in TVIX on 21 February 2012 having reached internal limits with the share price of TVIX, opening 90% higher than its $7.62 net assets. On 22–23 March 2012 the premium closed, with TVIX falling 30% for two consecutive days. The experience of the corresponding ETF, ProShares Ultra VIX Short-Term Futures ETF (UVXY) is also included in our study since its inception on 3 October 2011.

2.2 Summary of data

In this section we consider the daily data for the VIX Futures index and the S&P 500 VIX Futures index (SPVXSTR) against six VIX ETPs benchmarked to S&P 500 VIX Futures indexes (VXX, VIXY, TVIX, UVXY, XIV and SVXY), from the inception of each ETP. Adjusted daily closing prices of the three ETNs are shown in Figure 2, indexed to the range 0–100 to allow for comparison. Adjusted daily closing prices of the three corresponding ETFs are omitted from the figure, noting these are over 99% correlated with the ETNs since inception [6]. The return

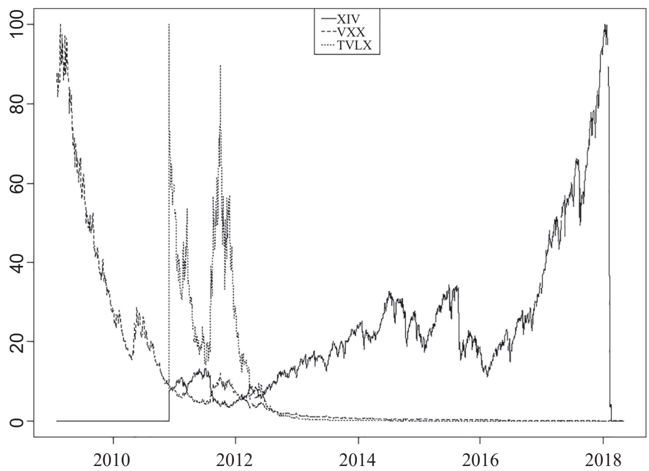


Figure 2.
Adjusted daily closing prices of the three ETNs – XIV, VXX and TVIX

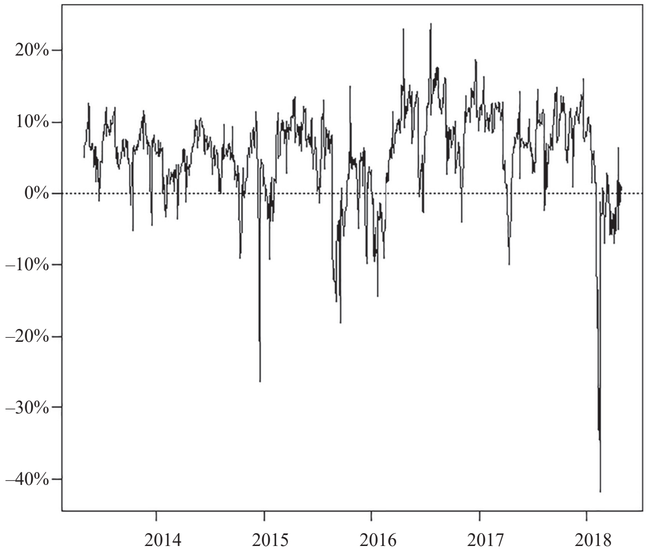
Note(s): This figure shows the Adjusted Daily Closing Prices of VIX ETNs are shown over the period January 2009 to March 2018. The adjusted daily closing prices of the three corresponding ETFs are omitted from the figure because these are over 99% correlated with the respective ETNs. Prices are indexed to the range 0 to 100 to allow for comparison

Source(s): Figure by authors

distributions of all ETFs studied are leptokurtic with a high probability for extreme values. VXX and TVIX have right skewed distributions as a result of the contango/roll-down effect vs the high returns when volatility spikes. The opposite holds for XIV.

Figure 3 also shows the term structure of VIX Futures over the period of analysis represented in terms of the price difference between the nearest and second nearest futures contracts, being the most liquid. The term structure tends to be inversely related to VIX, with the futures curve moving into backwardation when VIX spikes and traders adjust their positioning.

Bollen *et al.* (2017) documents the extraordinary movements in VIX over the period since VIX Futures were introduced in March 2004 to April 2013. They report the rising demand for tail risk insurance against falls in the stock market, particularly by retail investors who could not otherwise access VIX Futures and Options. They highlight the four distinct phases of growth since the launch of VIX Futures on 26 March 2004. Driven by immense retail demand for ETPs, open interest in ETPs outstripped that of VIX Futures early in 2010 (Phase 3). However, the ratio settled down to 20% of open interest 3 years later, with excess demand for ETPs versus VIX Futures no longer apparent (Phase 4). Figure 4 extends Phase 4 to February 2018. In fact, the end of the data series corresponds with the calmest year for the S&P 500 since 1965, with VIX recently hitting a twenty-year low. Since 2013, we also observe a low-volatility period similar to the mid-1990s and mid-2000s, with a higher frequency of short-lived volatility spikes where volatility jumps more than 50% in a one-week period and quickly reverts. In this paper, we consider the period since the end of the Bollen *et al.* (2017) studies as a phase of growth in VIX products markets worthy of further investigation. This was a period characterised by structurally low volatility and quick reversals, with global monetary policy contributing to low volatility, and assets under management in inverse and leveraged products at record highs. Over this entire period, the vega traded in VIX Futures and Options has remained ahead of SPX and SPY Options markets.



Note(s): This figure shows the Term Structure of Volatility. This is represented by the difference between second nearest and nearest VIX Futures contract prices as a percentage of the second contract price from April 2013 to March 2018. Negative values indicate backwardation and positive values indicate contango
Source(s): Figure by authors

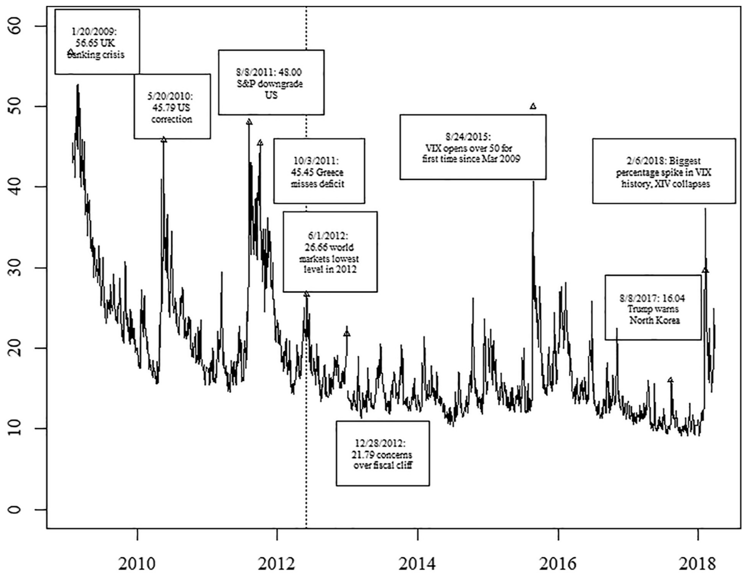
Figure 3. Term structure of VIX Futures

Phase 4 is a low volatility period and traders have increasingly focused on short-dated, near-the money trading in VIX options, and volatility selling strategies including inverse VIX ETPs. In 2017 when inflows in –1x inverse products were strongest, there were only eight 1% moves in the SPX and no 2% moves all year [7].

The growth in short volatility products and associated increase in rebalancing risk (gamma) clearly had the potential to exacerbate changes in volatility. The tail risks of inverse and levered long ETPs became evident on 16 February 2018 where XIV collapsed, and the leverage of SVXY and UVXY was cut on 27 February signalling the end of the recent low-volatility regime. Post February 2018, the AUM in short ETPs has fallen with the collapse of XIV and the leverage cut in ProShares SVXY and UVX, removing this "gamma overhang" for short-dated volatility.

3. Analysing price movements using high-frequency data

This section focuses on the causal relations between VIX ETPs and VIX futures prices. We use new methods well-suited for use with high-frequency and asynchronous data [8]. First, we study Granger causality in order to validate our understanding of price movements and look for variance in behaviour between products. We assess the elasticity of VIX futures price changes to VIX ETPs to highlight the rebalancing sensitivity of leveraged and inverse products. Next we move on to assess the lead-lag relations between VIX products. Analysis of lead-lag relations is motivated by the suspicion that not only do VIX futures and VIX ETPs predict VIX which is not a tradable asset, but there may also be situations where arbitrage relations between VIX futures and VIX ETPs break down. We categorize shifts in VIX



Note(s): This figure shows the Daily Closing Levels of VIX. The figure shows daily closing levels of VIX during the sample period March 2004 through March 2018. The study continues to March 2018, beyond a previous study by Author (2017) which ended in April 2013. Phase 3 begins on 29 January 2009, the launch date of the first VIX ETPs. Phase 4 begins on 29 February 2012, when VIX Futures trading races ahead of VIX options trading

Source(s): Figure by authors

Figure 4.
Daily closing levels of VIX

futures from contango to backwardation (and back) as likely times of market dislocation, and assess the duration of leads and lags.

3.1 Granger causality between VIX ETPs and futures

In our first investigation, we identify equilibrium Granger causality within a multivariate framework using intraday data. We use a Vector Error Correction Model (VECM) based on the technique developed by Yamamoto-Kurozumi (2006) [9]. Appendix contains a detailed description. We also consider the subsample of observations for each day between 3:45 p.m. and 4:15 p.m. on the suspicion that rebalancing activity of inverse and leveraged products might have a disproportionate impact on end-of-day VIX futures volatility.

The results presented in Table 1 are mixed. The strongest evidence of causality appears with the inverse products. We find evidence of bi-directional negative causality between VXX and XIV. Changes in VXX are associated with negative changes in XIV in 60% of the days in the full sample, and changes in XIV are associated with negative changes in VXX on 48.8% of the days. While these values are high in an absolute sense, we can gather greater clarity by comparing them with positive changes. Table 1 shows that changes in VXX cause positive changes in XIV only 5.7% of the time, and XIV causes positive changes in VXX only 6.0% of the time. Clearly, the negative causality is the most important. Interestingly, there is no evidence of causal relations between the corresponding ETFs, VIXY and SVXY. VIXY causes negative (positive) changes in SVXY 11.0% (10.0%) of the time in the full sample, and SVXY causes negative (positive) changes in VIXY 10.4% (10.4%) of the time. In the majority of the

Panel A – VXX, TVIX and XIV ETNs with SPVXSTER						
		Full sample			3:45–4:15 p.m.	
Sign of coefficient	−1	0	1	−1	0	1
Ln(VXX) → Ln(SPVXSTR)	11.7	75.1	22.1	17.6	63.6	27.7
Ln(VXX) → Ln(TVIX)	13.7	42.7	52.5	27.6	38.2	43.2
Ln(VXX) → Ln(XIV)	60.0	43.2	5.7	54.6	33.2	21.1
Ln(TVIX) → Ln(SPVXSTR)	16.0	78.9	14.0	23.5	65.9	19.5
Ln(TVIX) → Ln(VXX)	12.9	57.9	38.0	17.6	63.2	28.0
Ln(TVIX) → Ln(XIV)	25.6	53.8	29.5	26.0	63.9	19.0
Ln(XIV) → Ln(SPVXSTR)	17.9	76.2	14.8	34.6	49.8	24.5
Ln(XIV) → Ln(VXX)	48.8	54.1	6.0	48.2	46.0	14.7
Ln(XIV) → Ln(TVIX)	30.8	47.3	30.8	32.0	49.8	27.2
Ln(SPVXSTR) → Ln(VXX)	11.8	73.1	23.9	23.1	25.0	60.9
Ln(SPVXSTR) → Ln(TVIX)	22.5	65.9	20.5	45.6	24.4	38.9
Ln(SPVXSTR) → Ln(XIV)	19.2	69.6	20.4	47.6	24.0	37.5
Elasticity						
Full Sample: TVIX = 0.087, VXX = 0.57 and XIV = −0.22						
3:45–4:15 p.m.: TVIX = 0.03, VXX = 0.17 and XIV = −0.32						

Panel B – VIXY, UVXY and SVXY ETFs with SPVXSTER						
Sample		Full sample			3:45–4:15 p.m.	
Sign of coefficient	−1	0	1	−1	0	1
ln(SVXY) → ln(SPVXSTER)	24.7	50.9	24.4	30.8	46.2	23.0
ln(UVXY) → ln(SPVXSTER)	9.9	38.2	51.9	16.2	39.3	44.4
ln(VIXY) → ln(SPVXSTER)	21.5	60.6	17.9	21.1	49.2	29.7
ln(SPVXSTER) → ln(SVXY)	17.8	69.4	12.8	28.9	51.0	20.1
ln(UVXY) → ln(SVXY)	27.6	66.4	6.0	43.8	45.7	10.6
ln(VIXY) → ln(SVXY)	11.0	79.0	10.0	24.4	52.8	22.8
ln(SPVXSTER) → ln(UVXY)	4.9	64.4	30.7	10.3	53.6	36.1
ln(SVXY) → ln(UVXY)	23.9	70.9	5.2	33.5	54.2	12.3
ln(VIXY) → ln(UVXY)	6.3	78.9	14.8	15.4	59.6	25.0
ln(SPVXSTER-) → ln(VIXY)	10.9	75.9	13.2	18.7	56.6	24.7
ln(SVXY) → ln(VIXY)	10.4	79.2	10.4	25.0	55.1	19.8
ln(UVXY) → ln(VIXY)	5.5	77.2	17.6	12.1	53.0	35.2
Elasticity						
Full Sample: UVXY = 0.25, VIXY = 0.28 and SVXY = −0.19						
3:45–4:15 p.m.: UVXY = 0.12, VIXY = 0.16 and SVXY = −0.20						

Note(s): Panels A and B this table show the percentage of total number of days with significantly positive and negative coefficients for each pair using ETNs and ETFs, respectively. The model in Panel A is fitted in each day (n = 1,784) during which all ETNs traded concurrently, 29 November 2010 through 15 February 2018. The model in Panel B is fitted each day (n = 1,638) during which all ETFs traded concurrently, 3 October 2011 through 15 February 2018. The notation −1 denotes the presence of negative Granger causality, 0 denotes no Granger causality and 1 denotes positive Granger causality

Table 1.
Multivariate analysis of long-run granger causality

days of the sample, there is no clear causation in one direction or the other. The difference between the ETN and ETF results may be explained by the fact that the ETFs are less frequently traded, and detecting causal relations is more difficult, consistent with relationship between trading costs and market liquidity observed by Fernandez-Perez *et al.* (2018). The results for VIXY and SVXY in the last 30 min of trading are qualitatively similar to the full sample. The causality is ambiguous in the majority of days.

The twice levered results are also interesting. There is a moderate level of bidirectional positive causality between VXX and TVIX, with changes in VXX associated with positive changes in TVIX (38.0%), as well as the converse (52.5%). The causality results are similarly interpreted but weaker in the last 30 min of trading. The frequency falls from 38.0% to 28.0% for VXX and 52.5%–43.2% for TVIX, respectively. During the last 30 min, however, the rate of trading in TVIX is dramatically increased due to the rebalancing activity of levered and inverse ETPs. In this case, causality patterns are mitigated, largely as a result of all of the markets moving in unison. The evidence for the twice levered ETFs is much less dramatic than for the ETNs, with significant positive causality between VIXY and UVXY 14.8% of the time and UVXY and VIXY 17.6% of the time. In the last 30 min of trading, however, there is a marked increase in positive feedback. The relation between VIXY and UVXY goes from 14.8% to 25.0%, and the relation between UVXY and VIXY goes from 17.6% to 35.2%. At the end of the day levered and inverse product hedging activities are more prevalent.

The results for the twice levered/inverse products are somewhat different. TVIX and XIV have almost an equal number of cases of positive and negative causality, both in the full sample and in the last 30 min of trading. UVXY and SVXY, on the other hand, offer evidence bidirectional negative causality, particularly in the last 30 min of trading. For the full sample, UVXY causes negative changes in SVXY 27.6% of the time (compared with positive changes 6.0% of the time) and SVXY causes negative changes in UVXY 23.9% of the time (compared with positive changes 5.2% of the time). Again, for the levered and inverse products, the last 30 min of trading portends differential market behaviour.

The causality results between SPVXSTR and the six ETPs are potentially very important. The level of SPTXSTR is our proxy for VIX futures prices, and the link between the futures prices and the prices of the VIX ETPs should be strongly governed by arbitrage. Beginning first with VXX, the most active VIX ETN, we find VXX causes positive changes in SPVXSTR 22.1% of the time in the full sample and 27.7% of the time in the last 30 min of trading. On the other hand, VXX causes negative changes in SPVXSTR 11.7% of the time in the full sample and 17.6% of the time in the last 30 min of trading. While on balance this suggests positive causality, in the preponderance of days, there is no causality in either direction (75.1% in the full sample and 63.6% in the last 30 min of trading). Conversely, SPVXSTR causes positive (negative) changes in VXX 23.9% (11.8%) of the time in the full sample and 60.9% (23.1%) of the time in the last 30 min of trading. Indeed, in the last 30 min of trading, there are only 25.0% of the days in which causality is ambiguous. Positive causality wins the day. What this seems to suggest is that during the last 30 min of trading, VIX ETP issuers/market makers become consumed with rebalancing their futures positions at the end of the day in preparation that following trading day's trading.

The results for TVIX and XIV are different but equally interesting. First, again on the preponderance of days the direction of causality between changes in TVIX and SPVXSTR and XIV and SPVXSTR is ambiguous. In the full sample, for example, this applies on 78.9% of the days for TVIX and 76.2% of the days for XIV. In the last 30 min of trading, the numbers are 65.9 and 49.8%, respectively. Similarly, for the full sample, changes in SPVXSTR cause negative (positive) changes in TVIX 22.5% (20.5%) of the time, and changes in SPVXSTR cause negative (positive) changes in XIV 19.2% (20.4%) of the time. The results are ambiguous 65.9 and 69.6% of the time, respectively. Now we have a point of departure. In the last 30 min of trading, direction of the causality is anything but ambiguous. Changes in SPVXSTR cause negative (positive) changes in TVIX 45.6% (38.9%) of the time and negative (positive) changes in XIV 47.6% (37.5%) of time. While the direction is uncertain, perhaps confounded by substitution between products in different environments, its significance is not.

3.2 Elasticity of VIX futures to ETPs

We now turn to assessing the elasticity of VIX futures to VIX ETP prices using a long-run contemporaneous regression from the VECM. Table 1 provides a summary of the results of a regression of ln(SPVXSTR) on different ln(ETP) returns using the full sample of intraday 15 min log returns but ignoring overnight returns. Panel A contains the results using the period 29 November 2010 through 15 February 2018—the period in which all ETNs were traded concurrently. Although the estimated coefficients on small from an economic standpoint, they are significantly different from 0 at the 1% level. The coefficients Panel B contains the results using the period 3 October 2011 through 15 February 2018—the period during which all ETFs were traded concurrently. The model is fitted using all intraday data and end of day data for the last 30 min before close. As noted in the Granger causality test results, we have reason to believe that the rebalancing of levered and inverse products is concentrated at the end of days on which futures returns are abnormally high in one direction or the other. Since all variables are expressed as natural logarithms, we can interpret the coefficients as measures of elasticity. In Panel A, the coefficient on ln(VXX) in the regression of ln(SPVXSTR) on ln(TVIX), ln(VXX), and ln(XIV) is 0.567. This means a 1% VXX return is estimated to elicit a 0.567% SPVXSTR return. In addition, because the estimates are in levels and all four variables are cointegrated, the vector can be standardized to evaluate the long-run causal effects for all other combinations. The effect of ln(SPVXSTR) on ln(VXX) is 1/0.567 or 1.764%.

3.3 Lead–lag relations between VIX futures, ETPs and VIX

In this section, we look for evidence of break-downs in arbitrage relations between VIX futures and VIX ETPs by analysing bivariate lead–lag relations. It is well established that because VIX is not a tradable asset, VIX futures and ETPs predict VIX. Equally, because VIX ETPs have been such an important source of liquidity in VIX futures, there could be situations where arbitrage relations break down. We take the shift in VIX futures from contango to backwardation as a likely trigger for volatility traders to change their positioning, noting that it tends to be associated with a spike in VIX and abnormal returns in VIX futures. Finally, we assess the impact on lead–lag relations, and whether the change in duration of leads and lags between VIX products is economically significant.

The lead/lag ratio (LLR) measures whether the sum of squared correlations across all lags of variable X against Y is greater than the converse (see Hayashi and Yoshida, 2005; Huth and Abergel, 2014; Hoffmann *et al.*, 2013; Bollen *et al.*, 2017). We calculate the LLR for the SPVXSTR and VIX against each of the ETNs (VXX, XIV and TVIX) and the corresponding ETFs (VIXY, UVXY and SVXY) respectively. The LLR in levels can be interpreted as a long-run causal relation between SPVXSTR, ETPs and VIX, similar to the interpretation in our multivariate analysis above. We assess statistical significance at the 5% level using a simulation approach to generate critical values for the LLR (O'Neill and Rajaguru, 2020). This corresponds to the asymptotic value of ~2.10, being the same level as in the Bollen *et al.* (2017) study which also used high frequency ETP and SPVXSTR data.

Table 2 reports the number of observations in each phase which are statistically significant at the 5% level in both tails. The table demonstrates that all six ETPs lead VIX more often than not, regardless of whether the term structure is in contango (F2<F1) or backwardation (F1>F2). The lead–lag relations between ETPs and futures are less obvious. However, when the term structure moves to backwardation (F1>F2), ETPs lead more frequently, for example with VXX and UVXY leading on the majority of such days. That is, we find a statistically significant change in lead–lag relations associated with shifts in demand across VIX futures.

The LLR is a ratio of R^2 which can be misleading when correlations are very low. Thus, we also need to assess economic significance of leads and lags, in terms of the magnitude and duration. In Figure 5, we separate days with an LLR which is statistically significant at the 5% level and plot the average cross-correlation function. Examples are shown for (A) VIXY

ETP	VIX futures price curve	No. of days	ETPs lead futures $p < 0.05$	Futures lead ETPs $p > 0.95$	No. of days	ETPs lead VIX $p < 0.05$	VIX leads ETPs $p > 0.95$
VXX	contango	1967	15.6	28.0	1970	31.2	17.7
	backwardation	335	38.2	18.2	335	45.4	10.1
XIV	contango	1546	17.9	17.0	1542	46.4	9.0
	backwardation	243	35.4	10.7	243	59.3	2.9
TVIX	contango	1568	9.8	21.4	1566	35.2	12.3
	backwardation	265	38.9	9.4	265	67.2	4.2
VIXY	contango	1554	16.8	26.1	1554	61.5	5.5
	backwardation	265	34.7	14.7	265	80.0	1.5
SVXY	contango	1414	20.1	19.3	1414	59.9	5.0
	backwardation	199	33.7	12.1	199	63.8	3.5
UVXY	contango	1421	13.6	17.9	1421	41.0	12.7
	backwardation	206	51.9	5.8	206	53.4	5.8

Note(s): The table reports the number of days the VIX Futures price curve is in contango or backwardation. The table also reports p-values as the proportion of days with p-values in each tail of the distribution. If $p < 0.05$ on a given day, ETPs lead futures/VIX with a probability of 95%, and, if a $p > 0.95$, denotes futures/VIX leads with probability 95%. The table separately reports these p-values for days where the VIX Futures price curve was in contango/backwardation

Table 2. Summary of lead/lag analysis of VIX futures/cash with VIX ETPs from their inception through 15 February 2018

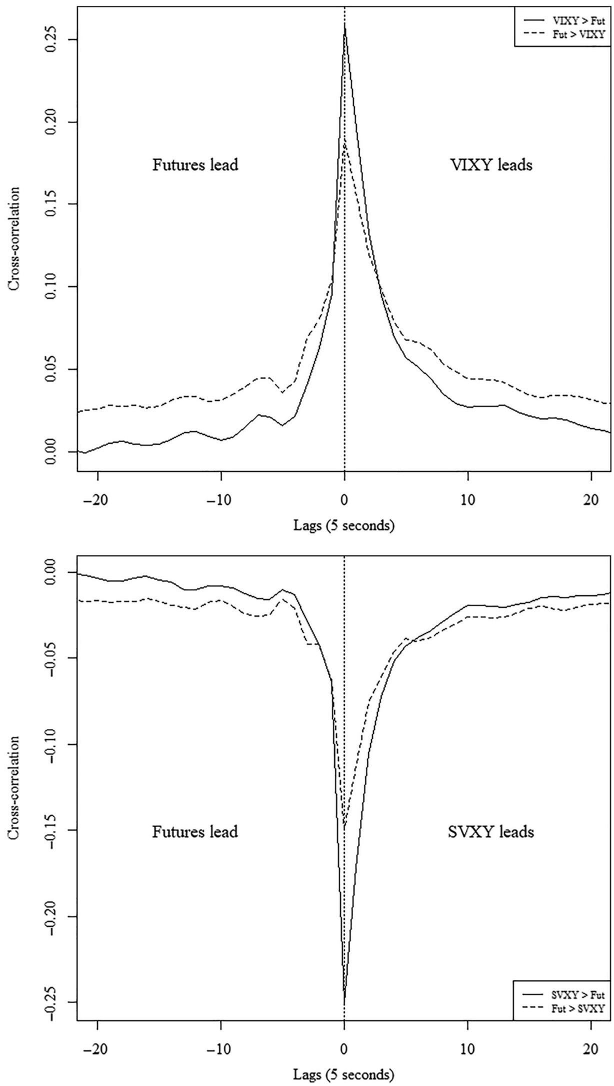
and (B) SVXY. We observe that cross-correlation functions generally spike more and are skewed to the right when ETPs lead and the LLR is significant. We also observe that where ETPs lead futures, the duration is at least 10 to 20 lags, or around 1–2 min. The duration is shorter for other ETPs such as TVIX, XIV and UVXY, persisting on average for 30–40 s, while leads and lags generally don't persist beyond 20 s for VXX. These non-contemporaneous relations suggest arbitrage opportunities might be possible. The converse is not true. When SPVXSTR leads, the function is much more symmetrical, cross-correlations generally have lower magnitude and the duration is less.

4. Conclusion

This paper studies causal relations between VIX ETPs and VIX Futures contracts. VIX product markets have grown to rival SPX and SPY options markets as the preferred venues for trading and hedging volatility. This growth in investor interest has coincided with changing causal relations between VIX Futures and ETPs over time. The causal relations between VXX and VIX Futures are well established with leads and lags generally found to be short-lived and arbitrage relations holding. We go further to capture 1x long, −1x inverse as well as 2x leveraged ETNs and the corresponding ETFs, to give a broad representation across the ETP market. We establish causal relations between inverse and leveraged products where causal relations are not yet documented.

Cointegration tests reveal unique stable long-run equilibrium relations between VIX ETPs and Futures. Our initial multivariate analysis of long-run Granger causality reveals the bidirectional long-run negative causality between 1x long ETN (VXX) and −1x inverse (XIV). Bidirectional positive causality exists between VXX and TVIX, but weakens during the last 30 min of trading, when rebalancing activity of levered and inverse ETPs increases dramatically and levered and inverse hedging demand is in the same direction. The corresponding ETFs which are less frequently traded show more ambiguous causality results.

Relations between SPXVSTR and ETPs are less clear. The causal relation between changes in ETPs (VXX, TVIX and XIV) and VIX Futures index (SPVXSTR) is ambiguous in both directions in the preponderance of days, except in the last 30 min of trading, where VIX



Note(s): This figure shows the cross correlation of leads and lags between ETPs and SPVXSTR. The cross-correlation function between ETPs and SPVXSTR is shown from May 2013 to March 2018, for days where lead-lag ratios were significant at the 95% level. The cross-correlation functions of SPVXSTR and VIXY is shown in (A), and SPVXSTR and SVXY in (B). Correlations are calculated using the Hayashi and Yoshida (2005) covariance matrix, which is generated each day after time-shifting each variable in increments of 5 seconds from 0 to 60 seconds. The corresponding vector of correlations has 121 elements. The average correlation at each lag is depicted

Source(s): Figure by authors

Figure 5.
Cross-correlation between ETPs and SPVXSTR

ETP issuers/market makers rebalancing activities are concentrated and SPVXSTR is significant in causing changes in ETPs.

Studies of high-frequency lead–lag relations reveal that all the 1x long, −1x inverse and 2x leveraged ETPs studied all lead VIX, regardless of whether markets are in contango or backwardation. As with VIX Futures, VIX ETPs also predict VIX. The lead-lag relations with VIX Futures are less obvious, similar to the findings of Bollen *et al.* (2017). However, we find that term structure of volatility has a significant impact on lead-lag relations between VIX Futures and ETPs. When the market is in backwardation, VIX ETPs tend to lead Futures more often, and particularly VXX and UVXY which lead in the majority of such days. Moreover, the duration of lead-lag relations can be 1–2 min when ETPs lead and lead–lag relations are statistically significant, suggesting that arbitrage opportunities might be possible.

Notes

1. Whaley (2013) attributes the rebalancing impacts of leveraged and inverse ETPs to the value of assets outstanding and the movement in VIX Futures. Park (2015) considers the volatility-of-volatility implied by VIX options and the implications for tail risk hedging. Fernandez-Perez *et al.* (2018) investigates intraday price discovery of VXX and XIV finding that informational leadership of the XIV increases on when VIX increases, and on days with negative stock market returns. They also find a relationship between trading costs and market liquidity.

2. Bansal *et al.* (2015) demonstrate that equity volatility serves as a determinant of future Treasury term-structure volatility in terms of level and slope. Mixon *et al.* (2019) decompose the VIX Futures term structure into systematic and idiosyncratic components, finding that the observed futures term structure is on average, steeper due to non-dealer demand.

3. During 2012, the term structure was steeply upward sloping leading to −97% returns for VXX. Had VXX been available in 2008, it would have earned returns of 214% in that year when the term structure was in backwardation.

4. Custodian and largest shareholder, Credit Suisse, announced that the product would be liquidated following the loss of 93% of its $1.9bn in asset value. Credit Suisse covered their position as a shareholder in XIV via the VIX Futures Market rather than selling XIV on market. They are now subject to a lawsuit (see https://www.cnbc.com/2018/03/19/credit-suisse-vix-etn-lawsuits-tidjane-thiam-says-bank-not-at-fault.html).

5. Tang and Shu (2013) examine why leveraged ETFs often have a different return than the leveraged multiple of the underlying return.

6. Since inception, VXX and TVIX have had four reverse splits, VIXY and SVXY three reverse splits, while UVXY had eight splits and XIV one split in the period shown.

7. With strong growth in short VIX ETPs, the rebalancing sensitivity (short gamma) of VIX Futures grew to record highs. When dealers are "long gamma" through leveraged and inverse products, they have to buy volatility when it is increasing and sell it when it is falling. They are effectively dampening realised volatility by selling equities when they rise and buying when they fall.

8. Traditional measures of causality rely on systematic sampling at regular time intervals and can produce spurious results when high-frequency data arrive at random times (Hayashi and Yoshida, 2005; Rajaguru *et al.*, 2018; Geweke, 1982; Rajaguru, 2004; Rajaguru and Abeysinghe, 2008). Rajaguru and Abeysinghe (2008) found that long-run cointegrating relations are preserved at all levels of aggregation and sampling intervals.

9. The Yamamoto–Kurozumi technique requires at least two of the variables to be non-stationary and are co-integrated. The unit root properties are examined through the Augmented Dickey–Fuller (ADF) test, the Phillips–Perron (PP) test, the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and the Dickey–Fuller Generalized Least Square (DF-GLS) unit root test. All variables are found to be non-stationary. The results are not reported here and can be made available upon request. The trace test and the maximum eigenvalue test are used to establish the number of cointegrating vectors. These results suggest one co-integrating vector, and hence there exists a stable unique long-run equilibrium relation between variables. The results are not reported here and can be made available upon request. The Tang and Shu (2013) procedure also support the robustness of the long-run Granger causality results and it can be made available upon request.

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Appendix

The Yamamoto–Kurozumi Technique

This co-integration test involves estimating the following bivariate, pth-order Gaussian vector autoregression (VAR) process

$$\begin{pmatrix} \ln(SPXVIXSTR_t) \\ \ln(TVIX_t) \\ \ln(VXX_t) \\ \ln(XIV_t) \end{pmatrix} = \mu + \sum_{i=1}^{p} \Pi_i \begin{pmatrix} \ln(SPXVIXSTR_{t-i}) \\ \ln(TVIX_{t-i}) \\ \ln(VXX_{t-i}) \\ \ln(XIV_{t-i}) \end{pmatrix} + \Theta D_t + \varepsilon_t, t = 1, 2, \ldots, T, \tag{1}$$

Since all four variables are co-integrated with one co-integrating vector, the following vector error correction model (VEC) is estimated to establish the long-run and short-run relationships between the variables.

$$\begin{pmatrix} \Delta\ln(SPXVIXSTR_t) \\ \Delta\ln(TVIX_t) \\ \Delta\ln(VXX_t) \\ \Delta\ln(XIV_t) \end{pmatrix} = \mu + \alpha e_{t-1} + \sum_{i=1}^{p} \Gamma_i \begin{pmatrix} \Delta\ln(SPXVIXSTR_{t-i}) \\ \Delta\ln(TVIX_{t-i}) \\ \Delta\ln(VXX_{t-i}) \\ \Delta\ln(XIV_{t-i}) \end{pmatrix} + \Theta D_t + \varepsilon_t, t = 1, 2, \ldots, T \tag{2}$$

where $e_t = (1 \quad \beta_1 \quad \beta_2 \quad \beta_3)\begin{pmatrix} \ln(SPXVIXSTR_t) \\ \ln(TVIX_t) \\ \ln(VXX_t) \\ \ln(XIV_t) \end{pmatrix}$ is an error process from the long-run static equation.

The main advantage of the Yamamoto-Kurozumi (2006) is the ability to identify the long- and short-run Granger causality separately. In order to determine the long-run Granger non-causality from the ith component of z_t to the jth component of z_t, we define two $1 \times n$ matrices, $R_L = [r_1 \quad r_2 \quad . \quad . \quad . \quad r_n]$ and $R_R^* = \left[r_1^* \quad r_2^* \quad . \quad . \quad . \quad r_n^*\right]$, such that $r_k = \{1 \text{ if } k = j|$ and $r_k = \{1 \text{ if } k = i|$.

For example, as specified in equation (1), to test long-run Granger non-causality from ln(*TVIX*) to ln(SPVXSTR), then $R_L = [1 \quad 0 \quad 0 \quad 0]$ and $R_R^* = [0 \quad 1 \quad 1 \quad 1]$. Long-run Granger non-causality from *ln(SPX)* to *ln(VIX)* is established by testing the null $H_0 : R_L\overline{B}R_R' = 0$. Specifically, we construct the Wald-type statistic using the generalized inverse given by

$$W^- = T\text{vec}\left(R_L\overline{\overline{B}}R_R'\right)'\left(R_L\widehat{C}\widehat{\Sigma}\widehat{C}'R_L' \otimes R_R\widehat{P}\widehat{\Sigma}\widehat{P}'R_R'\right)^{-g}\text{vec}\left(R_L\overline{\overline{B}}R_R'\right)\overrightarrow{d}\chi_s^2 \tag{3}$$

where T is the sample size, *vec* denotes the vectorization of a matrix by constructing a column vector from a matrix by appending each column of a matrix and $\widehat{\Sigma}$ is a consistent estimator of Σ, given by $\widehat{\Sigma} = T^{-1}\sum_{i=1}^{T}\widehat{\varepsilon}_t(\widehat{\beta})\widehat{\varepsilon}_t'(\widehat{\beta})$, where $\widehat{\varepsilon}_t(\widehat{\beta}) = [(\widehat{\beta}' z_{t-1})', \Delta z_{t-1}']'$. Also, $\overline{B} = \beta\beta' M' + \beta E_{12}(I - E_{22})^{-1}L'G'K^{-1}$, where β is a $n \times (n-1)$ matrix such that $\beta'\beta = 0$, $M = \begin{bmatrix} I_n \\ 0 \end{bmatrix}$, $E = \begin{bmatrix} I_{n-1} & \beta'_\perp\alpha & \beta'_\perp\Gamma_1 H \\ 0 & 1 + \beta'\alpha & \beta'\Gamma_1 H \end{bmatrix} = \begin{bmatrix} I_{n-1} & E_{12} \\ 0 & E_{22} \end{bmatrix}$, $G = I_2 \otimes H$, $H = [\beta, \beta]$, $L = \begin{bmatrix} 0 \\ I_{n+1} \end{bmatrix}$ and $K = \begin{bmatrix} I_n & 0 \\ I_n & -I_n \end{bmatrix}$. Further define the long-run impact matrix $C = \beta(\alpha'\Gamma\beta)^{-1}\alpha'$, where α is a $n \times (n-1)$ matrix such that $\alpha'\alpha = 0$ and $\Gamma = -(I + \Pi_2)$ and $P = Kr^{-1}GL(I_{n+1} - E_{22}')^{-1}\begin{bmatrix} I & 0 \\ 0 & I \otimes H' \end{bmatrix}$. Note that Q^{-g} denotes the generalized inverse of matrix Q and $s = \text{rank}(R_L\beta) \times \{\text{rank}(R_R\beta) + 1\}$

Similarly, the Yamamoto–Kurozumi framework can be applied to the following specification as well:

$$\begin{pmatrix} \ln(SPXVIXSTER_t) \\ \ln(VIXY_t) \\ \ln(UVXY_t) \\ \ln(SVXY_t) \end{pmatrix} = \mu + \sum_{i=1}^{p}\Pi_i\begin{pmatrix} \ln(SPXVIXSTER_{t-i}) \\ \ln(VIXY_{t-i}) \\ \ln(UVXY_{t-i}) \\ \ln(SVXY_{t-i}) \end{pmatrix} + \Theta D_t + \varepsilon_t, t = 1, 2, \ldots, T$$

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